Zeeman effect

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Introduction

The most important experimental technique triggering the development of modern atomic theory is spectroscopy, the observation of characteristic wavelengths of light that atoms can absorb and emit. A so-called wavelength dispersive element such as a water drop, a prism or a grating spatially separates different wavelengths of light emitted from one source. This is called a spectrum. If the light source emits white light, a band of all colours will be found after the light passes the wavelength dispersive element (e.g. rainbow). When observing the light from a source that contains only atoms of a certain element, one finds distinct lines instead of the complete band on the detector. The corresponding wavelengths are hence called "spectral lines"of the element. The additional information needed to be able to draw conclusions about the atomic structure from spectral lines was provided by the experimental observation of Rutherford and Planck. Rutherford found that in an atom, electrons surround a heavy, positively charged nucleus in the center. Planck developed an explanation of the blackbody radiation spectrum by quanta of light, photons, whose energy E_{ph} is connected with the wavelength λ by

$$E_{ph} = h\nu = \frac{hc}{\lambda} \tag{1}$$

where ν is the frequency, h is the Planck constant and c is the speed of light. With this knowledge it became clear that electrons in an atom can occupy only certain energy levels E_i, and that the transition of an electron from a higher energy level E_2 to a

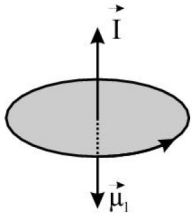

Figure 1: Classical model of an electron with angular momentum **l** inducing an angular magnetic moment $\vec{\mu}_l$.

lower energy level E_1 leads to the emission of a photon with energy $E_{ph} = E_2 - E_1$. The energy needed to lift the electron to the excited energy level can be provided by absorption of a photon of the very same energy or by collisions.
The increasing experimental precision reached with spectroscopy inspired scientists to propose different atomic models, leading to the development of quantum mechanics. With refinements to the quantum mechanical description of the atom, e.g. relativistic quantum mechanics (necessary to explain the splitting of energy levels of the same main quantum number n) and quantum electrodynamics (QED, necessary to explain the splitting of levels with the same quantum number j but different quantum number l), theory is by now able to make highly accurate predictions about the atomic structure.

Effects of an external magnetic field on the atomic structure

If the atom is placed in an external magnetic field, the atomic energy levels split and the observed spectrum changes. This was first observed by the dutch physicist Pieter Zeeman and is hence called the Zeeman effect. In an intuitive approach to explain the Zeeman effect, consider an atom with an electron with charge e moving with the speed v around the nucleus on a Bohr orbit with radius r. This orbiting electron can be described by a current

$$I = -e \cdot \frac{v}{2\pi r} \tag{2}$$

inducing an orbital magnetic moment

$$\vec{\mu}_l = I \cdot \mathbf{A} = I \cdot \pi r^2 \mathbf{n} = \frac{evr}{2}\mathbf{n} \tag{3}$$

where $\mathbf{A} = \pi r^2 \mathbf{n}$ is the area vector perpendicular to the orbit area πr^2. The angular momentum of the electron with mass m_e is

$$\mathbf{l} = \mathbf{r} \times \mathbf{p} = m_e \cdot r \cdot v \cdot \mathbf{n}. \tag{4}$$

An external magnetic Field **B** interacts with the angular magnetic moment and, hence,

changes the potential energy of the electron:

$$\Delta E_{pot} = -\vec{\mu}_l \cdot \mathbf{B} = \frac{e}{2m_e} \cdot \mathbf{l} \cdot \mathbf{B}. \tag{5}$$

Using the direction of the magnetic Field vector **B** as the quantization axis z and the quantized angular momentum of the electron

$$|\mathbf{l}| = \sqrt{l(l+1)}\hbar \quad \text{with} \quad l = 0, 1, \ldots, n-1 \quad \text{and} \tag{6}$$

$$l_z = m_l \cdot \hbar \quad \text{with} \quad -l \leq m_l \leq l, \tag{7}$$

Eqn. 5 can be simplified to

$$\Delta E_{pot} = \frac{e \cdot \hbar}{2m_e} \cdot m_l \cdot B = \mu_B \cdot m_l \cdot B, \tag{8}$$

where μ_B is called the Bohr magneton. The energy shift of the original energy level of the electron by ΔE_{pot} due to the coupling of the external magnetic field to the angular magnetic moment of the electron leads to a splitting of the original level into the previously degenerated $2l + 1$ sublevels of same angular momentum l, but different magnetic quantum number m_l.
The quantum mechanical way for deriving this energy splitting, where also the electron spin is taken into account, starts off by treating the external magnetic field as a perturbation $\hat{H}_B$ of the unperturbed Hamilton Operator $\hat{H}_0$. In general, an atom with more than one electron has to be considered. In the case of an external magnetic field being small compared to the magnetic coupling energies between the ith orbital and spin momenta of the electron $\mathbf{l}_i$ and $\mathbf{s}_i$, respectively, one may use the so-called *LS*-coupling approximation. This approximation is appropriate when the coupling energy between the different orbital momenta of the electrons is large compared to the coupling of a single orbital momentum of one electron with the same electron spin momentum. Hence, all orbital and spin momenta are added separately to total orbital

and total spin momenta **L** and **S** before determining the total angular momentum **J**:

$$\mathbf{L} = \sum_i \mathbf{l}_i \quad \text{with} \quad |\mathbf{L}| = \sqrt{L(L+1)}\hbar, \tag{9}$$

$$\mathbf{S} = \sum_i \mathbf{s}_i \quad \text{with} \quad |\mathbf{S}| = \sqrt{S(S+1)}\hbar, \tag{10}$$

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \quad \text{with} \quad |\mathbf{J}| = \sqrt{J(J+1)}\hbar, \tag{11}$$

$$J_z = M_J \cdot \hbar \quad \text{with} \quad -J \leq M_J \leq J. \tag{12}$$

With these momenta introduced, the Hamiltonian of an atom in an external magnetic field is

$$\hat{H}_0 + \hat{H}_B = \hat{H}_0 - \frac{\mu_B}{\hbar}(\mathbf{L} + \mathbf{S}) \cdot \mathbf{B} \tag{13}$$

and the solution for the shift of the level energy becomes

$$\Delta E_{pot} = \mu_B \cdot B \cdot M_J \cdot g_J \tag{14}$$

$$g_J = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, \tag{15}$$

with the Landé factor g_J.
If all electron spins in the atom cancel each other, ($S = 0$,) Eqn. 5 becomes identical to Eqn. 14 because $g_J = 1$.

For historical reasons, the splitting in absence of spin is called the *normal* Zeeman effect, because the electron spin was not yet discovered at that time.The more general case with spin ($S \neq 0$) is named the *anomalous* Zeeman effect. In the normal Zeeman effect, the size of the energy splitting from the unperturbed level energy depends only on the magnetic quantum number m_l. Therefore, it is of the same size for all levels in the atom. In the case of the anomalous Zeeman effect, the energy splitting depends on the quantum numbers J, L and S, which is reflected by more complicated structures observed in the corresponding spectra (Fig. 2). In a strong external field, the LS-coupling approximation is no longer appropriate. The field-induced precessions are so rapid that the total orbital momentum L and spin S have to be considered separately, because they individually precess around **B**. This means that L and S are decoupled and J is rendered meaningless. This is called the Paschen-Back effect.

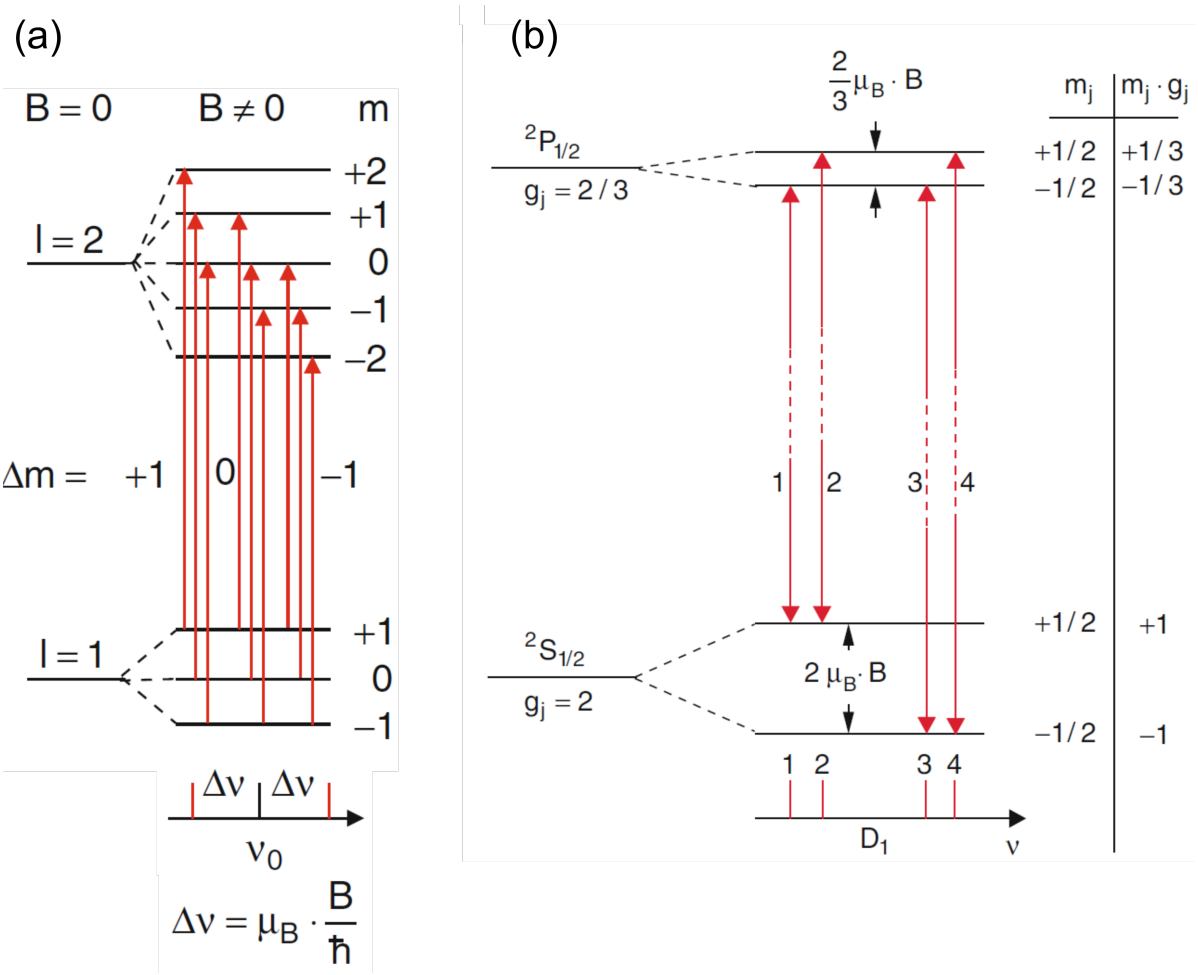

Figure 2: Left: Normal Zeeman effect for a p $\longrightarrow$ d transition. The field splits the degenerate M_J levels by $\Delta E = \mu_B \cdot B \cdot M_J$. Right: Anomalous Zeeman effect. Here, the levels split by $\Delta E = \mu_B \cdot B \cdot M_J \cdot g_J$. In both cases, transitions can only occur according to the selection rules. (Source: Experimentalphysik 3, W. Demtröder, 2015)

Selection rules and polarization of emitted light

In transitions from an excited level i to a lower level k, a photon with the energy $E_{ph} = \Delta E = E_i - E_k$ is emitted. Still, as indicated in Fig. 2, not all energy differences between existing levels are actually found in the spectra. The reason is that not only the energy, but also momentum conservation, angular momentum conservation and symmetry rules play a role on whether or not a transition can occur. Only those transitions are allowed where the transition dipole matrix element

$$\mathbf{M}_{ik} = e \int \Psi_i^* \, \mathbf{r} \, \Psi_k \, dV \tag{16}$$

has at least one non-zero component

$$(M_{ik})_q = e \int \Psi_i^* \, q \, \Psi_k \, dq, \quad \text{with } q = x, y, z. \tag{17}$$

Evaluating these three integrals yields that only those transitions have non-zero components where the change of the magnetic quantum number fulfills $\Delta M_J = M_{J,i} - M_{J,k} = 0, \pm 1$. Furthermore, electric dipole transitions require that the change of the total orbital momentum is $\Delta L = L_i - L_k = \pm 1$ and that the spin does not change, i.e. $\Delta S = 0$. If the quantisation axis (i.e. the direction of the magnetic field vector) is the z-axis, the matrix element component $(M_{ik})_z$ is the only non-zero one for $\Delta M_J = 0$ (socalled 'π'-transitions). Considering a dipole and its characteristics of emission, this means the dipole is oscillating along the axis of the magnetic field vector and, hence, does not emit any radiation in that direction. Also, since the electric field vector is oscillating in only one direction, also the field vector of the emitted light wave oscillates in this direction, meaning that the light emitted by $\Delta M_J = 0$ transitions is linearly polarised (see Fig. 3). In the case of $\Delta M_J = \pm 1$('σ'- transitions), $(M_{ik})_z$ is zero and both $(M_{ik})_x$ and $(M_{ik})_y$ are non-zero and equal in magnitude, but phase-shifted by $\pi/2$. This phase shift between the two superimposed dipoles leads to the observation of circularly polarised light when viewing in z-direction ('longitudinal'). Viewing the emitted light in x- or y-direction ('transversal'), only the projection of the superimposed dipoles is seen, and thus the light from $\Delta M_J = \pm 1$ transitions appears linearly polarised when observed transversal to the magnetic field vector.

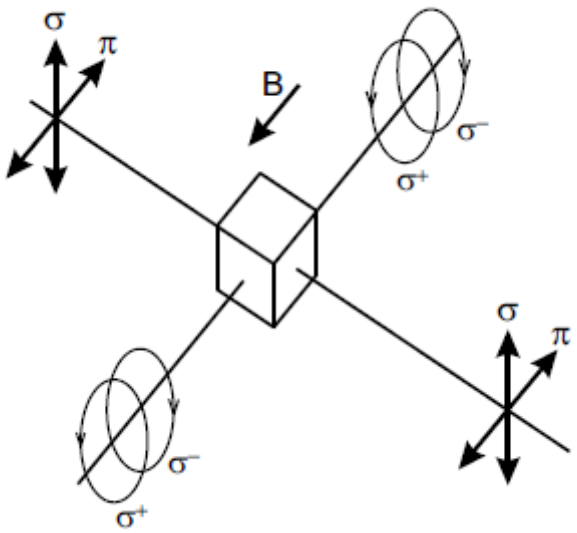

Figure 3: Isometric depiction of the polarisation of the different Zeeman components.

The Experiment

The experiment consists of two parts. In the first part, the Zeeman effect in transverse and longitudinal directions is investigated, and the polarisation of the observed lines are examined. The second part is dedicated to precision spectroscopy with a grating spectrometer. In both parts of the experiment the light emitted from a Cadmium lamp (Cd) will be used. The electronic configuration of Cadmium is $[\mathrm{Kr}]4d^{10}5s^2$. In this configuration all spins of the shell electrons compensate for each other, such that $S = 0$. In visible light, the Cd spectrum consists of four lines, given in Tab. 1. All measurements will be made on the red line, $\lambda_0 = 643.8\,\mathrm{nm}$ $(\nu_0 = 465.7\,\mathrm{THz})$, which arises from the transition between the two excited levels $^1\mathrm{D}_2 \rightarrow ^1\mathrm{P}_1$ in the fifth shell of Cd.

Table 1: Spectral lines of Cadmium in the visible range.

Colour	violet	blue	green	red
Wavelength (nm)	467.9	480.1	508.7	643.8

Part I: Spectroscopy of the Zeeman Effect

Since the expected relative shift in wavelength due to the external magnetic field is smaller than 0.1 nm, a very high spectral resolution is required to resolve the effect. In this experiment, a so-called Lummer-Gehrcke plate will be used (Fig. 4, 5). It consists of a quartz glass plate with highly plane-parallel surfaces.

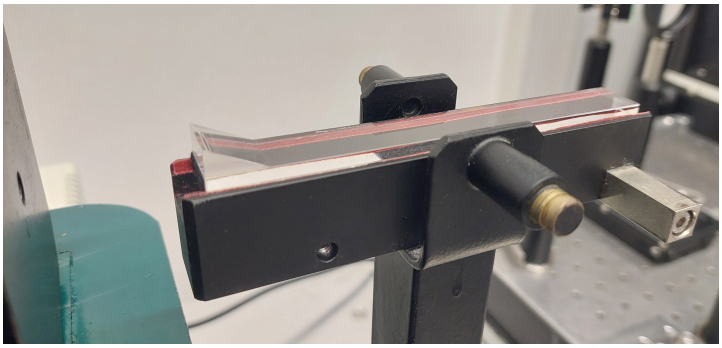

Figure 4: Photo of the uncovered Lummer-Gehrcke plate

Light enters the plate via a prism and is then reflected up and down between the inner surfaces at an angle close to total internal reflection. At each reflection, a small

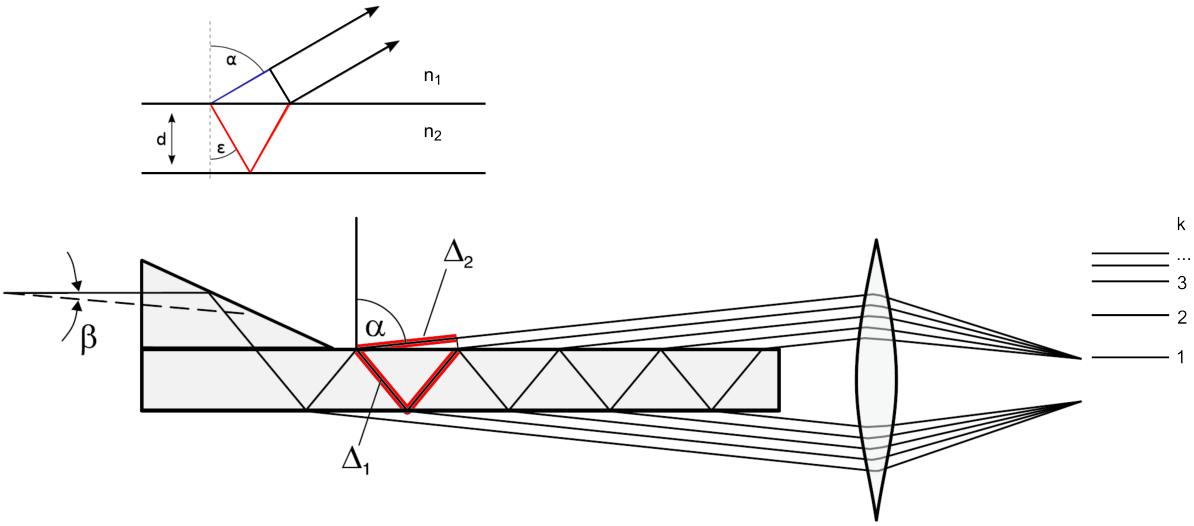

Figure 5: Schematic of the Lummer-Gehrcke plate. Note: In this figure, n_1 refers to the surrounding medium (typically air), and n_2 to the plate material.

portion of the light escapes the plate and can be observed using focusing optics. Due to the plate's length being much greater than its thickness, and because of the high reflectivity, many beams can interfere, resulting in very high resolution. Different angles of incident result in different orders of interference, marked as k in Fig. 5. The optical path difference between two beams exiting the plate, one of which is delayed by a round trip within the glass plate, is given by

$$\Delta = 2n_2 \cdot \frac{d}{\cos(\epsilon)} - 2n_1 \cdot d \cdot \tan(\epsilon) \cdot \sin(\alpha), \tag{18}$$

where d is the thickness of the plate, n_1 and n_2 are the refractive indices outside and inside the plate, and α and ϵ are the external and internal angles of the light rays with respect to the plate normal. Using Snell's law, $\sin(\alpha) = \frac{n_2}{n_1} \cdot \sin(\epsilon)$, and the identity $\sin^2(\epsilon) + \cos^2(\epsilon) = 1$, Eq. (18) simplifies to

$$\Delta = 2n_2 \cdot d \cdot \cos(\epsilon) = 2d \cdot \sqrt{n_2^2 - n_1^2 \sin^2(\alpha)}. \tag{19}$$

For a plate in air ($n_1 \approx 1$) and for incident light at nearly 90°, Eq. (19) reduces to

$$\Delta = 2d \cdot \sqrt{n^2 - 1}, \tag{20}$$

with $n := n_2$. The path differences Δ_1 and Δ_2 are illustrated in Fig. 5. Constructive

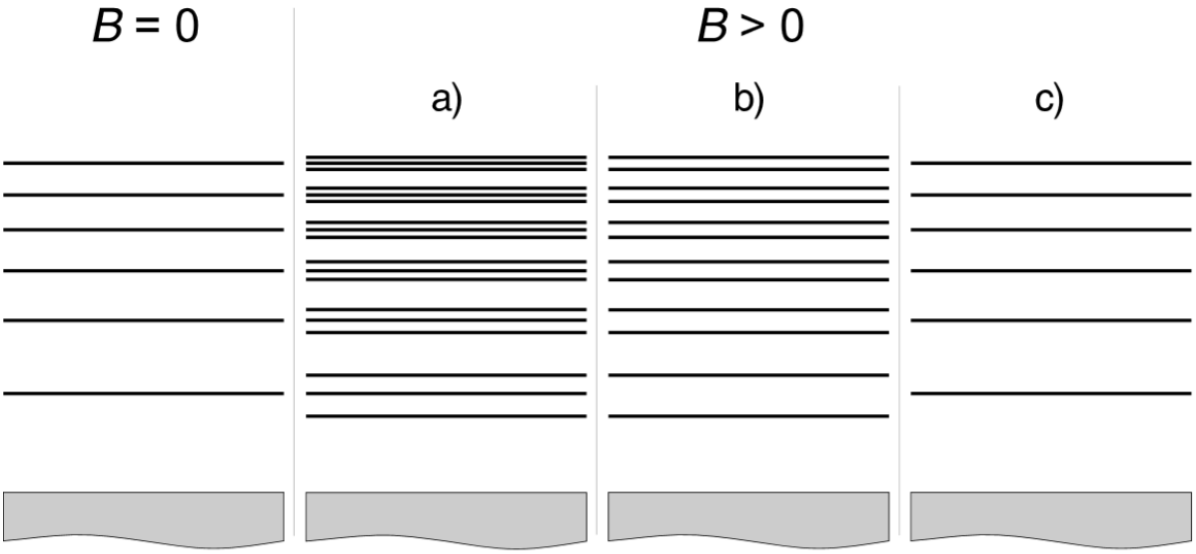

Figure 6: Interference pattern of the Zeeman effect, observed in the transverse direction: (a) without a polarization filter, (b) with the polarization axis perpendicular to the magnetic field, and (c) with the polarization axis parallel to the magnetic field.

interference occurs when the optical path difference satisfies the condition

$$\Delta = k \cdot \lambda,$$

for integer values of k and wavelength λ. Whether constructive or destructive interference occurs depends on the wavelength λ, the emission angle α, and therefore also on the incidence angle β of light entering the plate. For a fixed wavelength λ, multiple interference orders k can occur for different values of β, as illustrated in Fig. 6. Two interference orders overlap spatially when the kth order of wavelength $\lambda + \Delta\lambda$ coincides with the $(k+1)$th order of wavelength λ:

$$k \cdot (\lambda + \Delta\lambda) = (k+1) \cdot \lambda \quad \Rightarrow \quad \Delta\lambda = \frac{\lambda}{k}. \tag{21}$$

Combining this with Eq. (20), the free spectral range (FSR) of the Lummer-Gehrcke plate is

$$\Delta\lambda = \frac{\lambda^2}{2d \cdot \sqrt{n^2 - 1}}. \tag{22}$$

A small wavelength shift $\delta\lambda \ll \Delta\lambda$ leads to a spatial shift of the interference pattern:

$$\delta\lambda = \frac{\delta a}{\Delta a} \cdot \Delta\lambda, \tag{23}$$

where δa is the shift between two lines of slightly different wavelength, and Δa is the fringe spacing between two adjacent orders of the same wavelength. Therefore, for small wavelength shifts as caused by the Zeeman effect:

$$\delta\lambda = \frac{\delta a}{\Delta a} \cdot \frac{\lambda^2}{2d \cdot \sqrt{n^2 - 1}}. \tag{24}$$

Czerny-Turner Spectrometer

The Czerny-Turner spectrometer splits up light into discrete wavelengths by a diffraction grating. The spectrum appears in the focal plane of the exit and is recorded there by an image sensor. As shown in Fig. 7, a curved mirror (C, "collimator mirror") is used to collimate the light rays, which reach the spectrometer through the entrance slit (B). After the light is diffracted at the grating, it is focused by another curved mirror (E, "camera mirror") onto the image sensor (F).

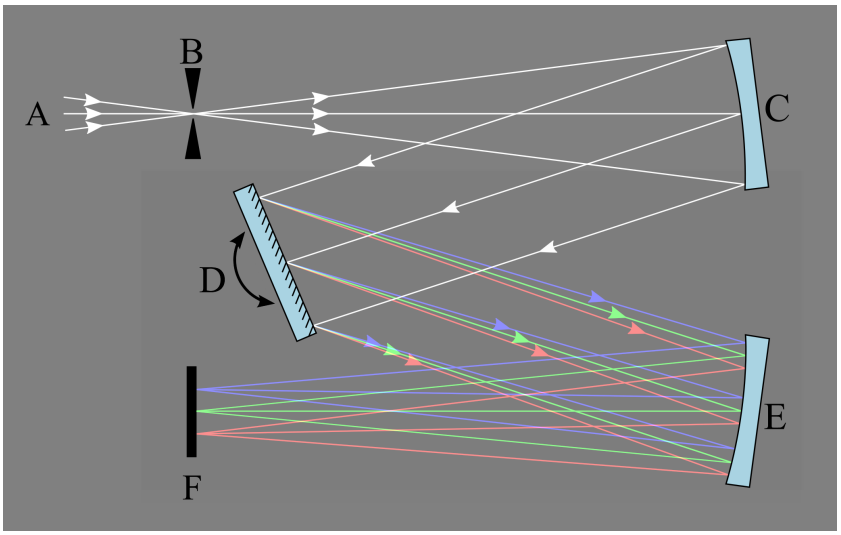

Figure 7: A diagram of a Czerny-Turner monochromator. Light (A) is focused onto an entrance slit (B) and is collimated by a concave mirror (C) with focal length f. The collimated beam is diffracted from a rotatable grating (D) and the dispersed beam re-focussed by a second mirror (E) with the same focal length f at the camera (F). Each wavelength of light is focused at a different position at the camera. (Adapted from `wikipedia.org`)

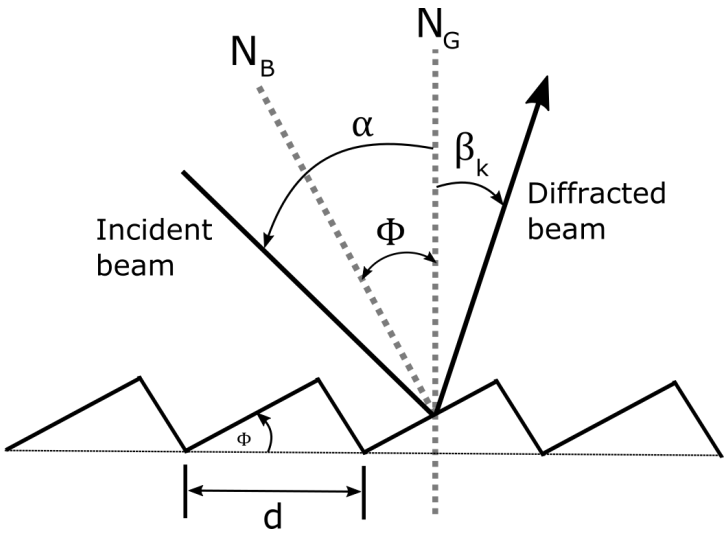

Figure 8: Schematic diagram of the reflection of a photon ray after interacting with a grating. Φ defines the *blaze angle*. The distance between consecutive lines is given by d.

Grating Characteristics

At a grating, the angles of the incoming and outgoing light rays with wavelength λ satisfy the following relationship:

$$\sin\alpha + \sin\beta_k = kn\lambda \tag{25}$$

with the angle of incidence α, the diffraction angle β_k relative to the normal (N_G) of the grating, the diffraction order k and the line density n. The line density corresponds to the number of grating lines per length. It is the reciprocal of the grating constant d which is the distance between two neighbouring lines of the grating. When $k = 0$, then Eqn. 25 reduces to $\alpha = -\beta_0$, the law of reflection. If the diffraction angle is assigned a fixed value, then the difference between α and β_k, the so called deflection angle (D_V), is constant (Fig. 8). The grating can concentrate a large portion of the diffracted radiation into one spectral order k. Hence the intensity of the other orders decreases. This redistribution of the intensity between the orders depends on the angle between the reflective elements and the grating surface, the so called *blaze angle* Φ (Fig. 8). In the visible spectrum the red region of the spectrum is diverted the least and the violet region the most.

Dispersion

The derivation of the diffraction angle after the wavelength is also referred to as *angular dispersion*. It is a measure of the angular distance between light beams with adjacent wavelengths. Through derivation of Eqn. 25 with respect to the wavelength λ for a fixed angle α, the following expression for the angular dispersion is obtained (Eqn. 27):

$$\frac{\partial \left(\sin \beta_k \left(\lambda\right)\right)}{\partial \lambda} = \cos \beta_k \, \frac{\partial \beta_k}{\partial \lambda} = kn \tag{26}$$

$$\Rightarrow \frac{\partial \beta_k}{\partial \lambda} = \frac{kn}{\cos \beta_k}. \tag{27}$$

A high angular dispersion can be achieved if either a grating with a high line density n or a coarse grating in high diffraction orders k is used. Most of the time it is more useful to describe the disperion in terms of distance p on a detector rather than in terms of angular distance β_k. Therefore, one calculates the *linear dispersion* from the angular dispersion:

$$\frac{\partial p}{\partial \lambda} = \frac{\partial p}{\partial \beta_k} \, \frac{\partial \beta_k}{\partial \lambda} \tag{28}$$

The term $\partial p / \partial \beta_k$ is the change of position on the detector for varying diffraction angle. The distance between the grating and the detector is the focal length f. Therefore, it is for small diffraction angles β_k:

$$\sin \beta_k \approx \frac{p}{f} \approx \beta_k \tag{29}$$

The derivation of the detector position with respect to the diffraction angle is therefore

$$\frac{\partial p}{\partial \beta_k} = f \tag{30}$$

which mans for the linear diffraction:

$$\frac{\partial p}{\partial \lambda} = f \, \frac{\partial \beta_k}{\partial \lambda} = \frac{fkn}{\cos \beta_k}. \tag{31}$$

Often, this formula is inverted to give the *inverted linear dispersion*:

$$D(\lambda) = \frac{\partial \lambda}{\partial p} = \frac{\cos \beta_k}{fkn}. \tag{32}$$

As the size of the measuring apparatus depends on the focal length of the optical system, it can be designed more compactly by using a grating with a higher line density. As shown in Eqn. 32, the relation between p and the wavelength λ is given by the dispersion function $D(\lambda)$. This function depends on the wavelength. At least one reference point p_0 in the spectrum (at a known wavelength λ_0) is necessary in order to calibrate the wavelength scale, if $D(\lambda)$ is known. Then the value λ of every other line in the spectrum can be obtained through

$$\lambda = \lambda_0 + \int_{p_0}^{p} D(\lambda) dp. \tag{33}$$

This is often numerically solvable. Though if the dispersion function is independent of λ, then Eqn. 33 can be expressed as following

$$\lambda = \lambda_0 + D(p - p_0), \tag{34}$$

and the desired wavelength λ can be determined. In this experiment reference lines should be used in order to find the dispersion function of the spectrometer. With the positions of the reference lines in the detector p_i and their wavelengths λ_i, the dispersion can be approximated by a polynomial function $\lambda(p)$.

$$\lambda(p) = A + B \cdot p + C \cdot p^2 + \ldots, \tag{35}$$

with the free parameters A, B, C, etc.

Resolution

The *resolution* or *chromatic resolution* of a grating describes its ability to distinguish neighbouring spectral lines. The resolution is defined as $R = \lambda/\Delta\lambda$, with the wavelength difference $\Delta\lambda$ between two spectral lines of the same intensity, that can be distinguished. The limit of resolution of a grating is $R = kN$, with the number N of the illuminated lines of the grating. An appropriate expression for the resolution is obtained through the use of Eqn. 25:

$$R = kN = \frac{N(\sin\alpha + \sin\beta_k)}{n\lambda} \tag{36}$$

Camera Systems

This experiment uses two different cameras:

- Thorlabs CS165CU/M Zelux camera (Colour CMOS camera, 1440 x 1080 pixels (1.6 MP), pixel size 3.45 µm x 3.45 µm, wavelength range 400 - 1000 nm), depiction of the intermediate image after Lummer-Gehrcke plate, this camera is only needed if beam path is misaligned
- Princton Intruments Pixis 2KBUV camera (CCD camera, 2048 x 512 pixels (1 MP), pixel size 13.5 µm x 13.5 µm, wavelength range 120 - 1100 nm), spectrometer camera

A CMOS camera, also called active pixel sensor, is an arrangement of semiconductor photodetectors with integrated amplifiers. The fact that each photodetector (pixel) has its own amplifier distinguishes active pixel sensors from other types of image sensors such as CCD cameras. The pixels convert the incoming light into electrons through the internal photoelectric effect. These electrons are saved in the pixels, amplified and read out by the integrated camera electronics. In this way, the sensor provides information about the light intensity and position at the same time.
The term CMOS refers to the the most commonly used manufacturing technique of active pixel sensors. This method uses complementary metal-oxide-semiconductors (CMOS) to create image sensors, hence the name for that type of camera.
A CCD camera (Charge-Coupled-Device) also consists of a matrix of photodiodes using the photoelectric effect to convert the light into an electrical charge that is shortly stored inside the pixel. Different to an active pixel system, a CCD camera uses a readout register: charges are transfered from pixel to pixel downwards along the columns to a readout register at the edge of the sensor. Then one charge packet at a time is transferred to an output amplifier where the charge is converted to a voltage signal.
Until the early 2000s, CCD sensors were mainly used in cameras. Nowadays, CMOS sensors have improved significantly in terms of technology, replacing CCD sensors in many applications (high readout speed, low power consumption). However, CCD cameras still offer better image quality for extremely low-light applications such as spectroscopy (due to their low noise level).

Experimental Setup

A schematic of the experimental setup is shown in Fig. 9. The electromagnet is placed right in front of the holder of the Lummer-Gehercke plate. A cadmium lamp is placed between the pole pieces of the electromagnet. The emitted light is focussed into the Lummer-Gehercke plate by lens 1 that is inserted into the holder (two different types of lens 1 available: one lens on a transparent base plate and one lens on a red filter plate to isolate the red Cd line, in normal student operation mode only the lens on the transparent base plate is needed). The magnet can be rotated by 90° to allow observations in both transverse and longitudinal configurations. The setup is designed in a way that the magnet can be moved and rotated independently from the Lummer-Gehrcke plate and the beam path. In order to rotate the magnet, it needs to be moved away some centimeters from the holder of the Lummer-Gehrcke plate. The longitudinal configuration is observed through a hole in one of the pole pieces.
The interference orders from the plate are focused to the intermediate image 150 mm behind lens 2. The intermediate image is enlarged by a factor of 2 and projected onto the entrance slit of the spectrometer using lens 3. To analyze the polarization of the light, a polarization filter and a $\lambda/4$ waveplate can be placed onto the output of the plate-holder.
To calculate the correct position and focus length of lens 3, to project a magnified image of the interference pattern onto the entrance slit, it is very helpful to apply the lens equation.

$$A = \frac{B}{G} = \frac{b}{g} \quad or \quad \frac{1}{b} + \frac{1}{g} = \frac{1}{f} \tag{37}$$

where A is the magnification, B and G are image and object sizes, b is the image distance, g is the object distance, and f is the focal length of the lens. A magnification of approximately 2 is needed in this experiment to achieve optimal resolution on the spectrometer camera.

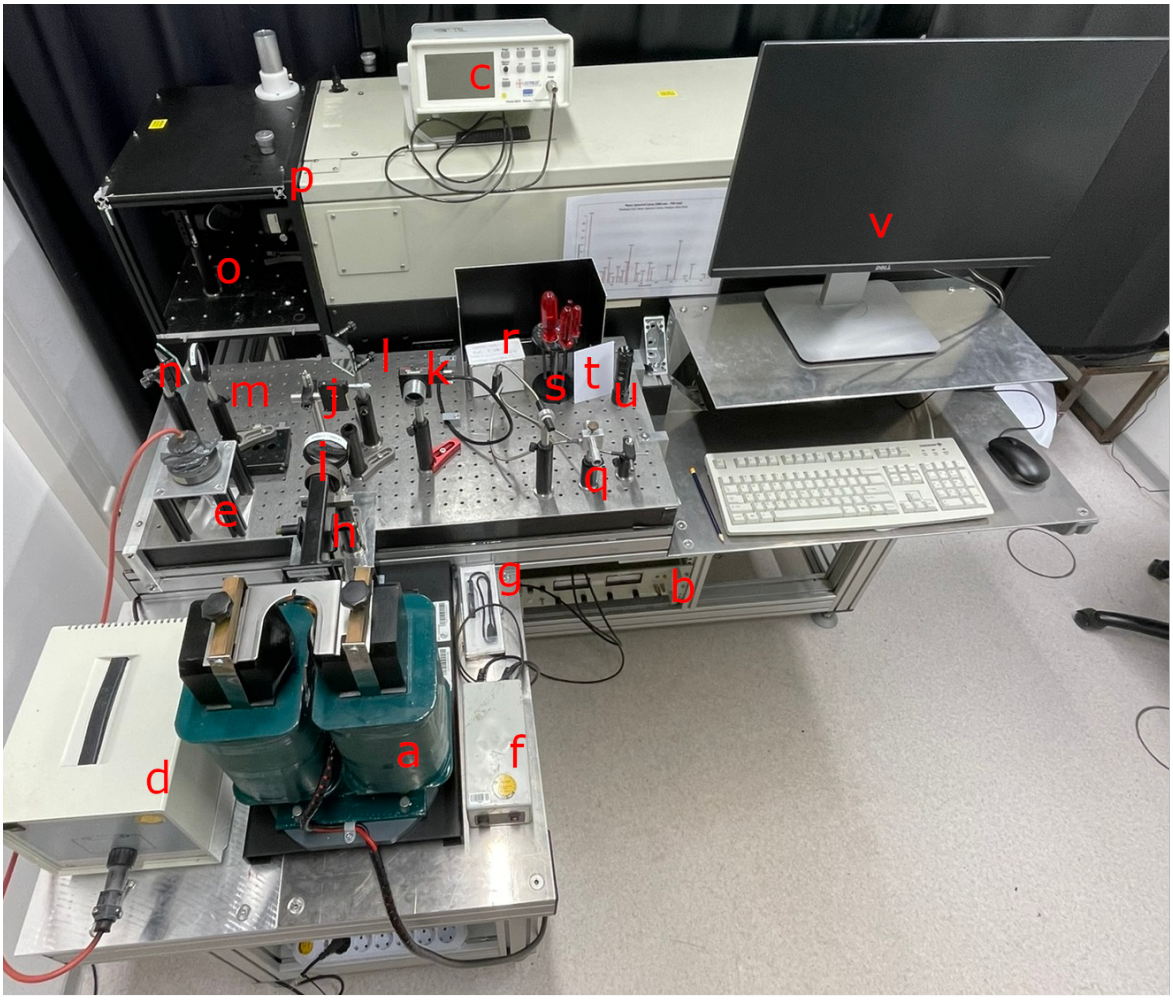

Figure 9: Experimental setup for investigating the normal Zeeman effect a) electromagnet with pole pieces, b) power supply for electromagnet, c) Hall probe, d) power supply Cd lamp, e) Cd lamp, f) power supply Ne lamp, g) Ne lamp, h) Lummer-Gehrcke plate with lens 1 and polarization filter mount, i) lens 2 (f=150 mm), j) slit 1, k) CMOS-camera, l) mirror 1, m) lens 3 (f=175 mm), n) mirror 2, o) mirror 3, p) entrance slit of Czerny-Turner spectrometer, q) fiber splitter, r) red filter and polarization optics, s) Allen keys, t) viewer card, u) flash light, v) experiment control PC

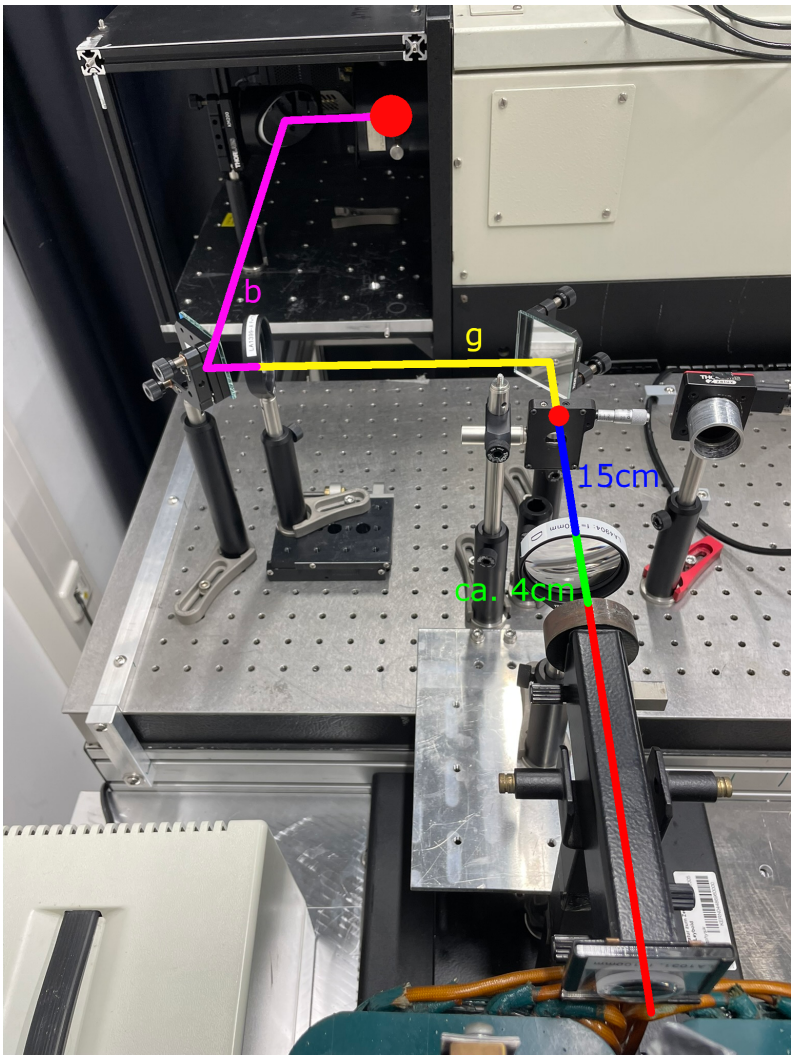

Figure 10: Optical beam path for imaging the interference orders of the Lummer-Gehrcke plate onto the entrance slit of the spectrometer. The red line indicates light emitted from the cadmium lamp, which is focused by lens 1 before entering the Lummer-Gehrcke plate. The green line marks the distance between the plate and lens 2. The blue line represents the distance from lens 2 to the intermediate image (indicated as a small red dot) formed behind slit 1. The yellow line (denoted as g) is the distance between the intermediate image and lens 3. The purple line shows the distance from lens 3 to the entrance slit of the spectrometer, where the enlarged image (large red dot) of the intermediate pattern is projected.

Part I: Execution

Security Advice

- The Lummer-Gehrcke plate and its cover may not be removed. The plate is very expensive!
- Always turn off the current before inserting or removing the Cd lamp.
- Never touch the Cd lamp with bare fingers. Moisture and grease on the glass reduce the lamp's lifetime — and the lamp gets very hot. Handle it carefully.
- Only move and rotate the magnet when the current is switched off.
- CAUTION: The magnet is very heavy (50 kg) - do not try to raise it, just slide it on the surface!
- Never set the the spectrometer grating to values <5 nm. It probably get stuck otherwise.

- **Use the teslameter (Hall probe) to measure the magnetic field strength** at the position of the Cd lamp between the pole pieces. Record the magnetic field as a function of the coil current between $I = 0\,\mathrm{A}$ and $I = 13\,\mathrm{A}$.
  Perform the measurements twice: once with increasing current and once with decreasing current, in order to observe the **hysteresis effect**. For each current value, take three measurements and calculate the average. Ensure that the power supply is set correctly (coarse voltage rotary knob to maximum, vary coarse current rotary knob carefully - do not exceed the limits for voltage and current marked on the power supply).
- **Start in transversal arrangement** of the magnet as shown in Fig. 9.
- **Carefully mount the Cd lamp** in its holder while the magnetic field is turned off. Avoid applying any force to the lamp. Turn on the Cd lamp and let it warm up for several minutes to reach full brightness.
- Open the **spectrometer entrance slit as wide as possible** by turning the knob above the slit.
- The distance between the intermediate image and the entrance slit is 78 cm. **Calculate the position of lens 3** to gain a magnification of factor 2 and reposition

the lens with the help of the adjusting screw of the movable stage. (*Note* that the position of the intermediate picture is at the position of the empty post holder behind the slit.)

- **Before you start measuring the interference pattern make sure that you read and followed the instructions "Operating The SPEX-Spectrometer" in the Appendix!!**
- **Record images of the interference pattern** with approximately five well separated interference orders (k):
  - without magnetic field
  - images of the σ and π Zeeman components at four different magnetic field strengths, corresponding to coil currents between $I = 9\,\mathrm{A}$ and $I = 13\,\mathrm{A}$, each with:
    - fully open entrance slit
    - very narrow entrance slit (interference stripes appear as little bullets).
- Use "repeated aquisition" mode of the camera as well as the polarization filter and the $\lambda/4$ waveplate (identify how to distinguish between the two) to **analyze the polarization of the different Zeeman components**. Also analyze how the spectrometer grating is acting like a polarization filter.
- **Repeat all measurements and observations in longitudinal arrangement.** Switch off the magnetic field and the Cd-lamp before moving and turning the magnet. Your tutor will show you how to rotate the magnet to change between the two geometries.

Part I: Evaluation

- **Plot the magnetic field strength as a function of the current.** Evaluate the presence and magnitude of the hysteresis effect. **Fit a linear function** $B(I)$ and use it to determine the magnetic field strengths corresponding to the current values you used.
  Note: If the fit does not match your data well, consider using the measured values directly and discuss possible reasons why a linear approximation may not apply.

- Record your **qualitative observations** of the Zeeman effect in both transverse and longitudinal configurations. Save the corresponding images for inclusion in your report.

- When **using the polarization filter and the $\lambda/4$ waveplate**, take note of which lines appear, under which conditions, and why. Are there any other polarization-sensitive elements in the setup? Discuss how the signal quality could be improved.

- In the **transverse configuration: Determine the positions a of all σ and π lines for all interference orders** from your recorded graphs. This should be done for each of the four measurements ($I = 9\,\mathrm{A}$ to $I = 13\,\mathrm{A}$). The lines may appear slightly tilted — why might this be the case? Correct for this by tilting the raw picture until the interference stipes are horizontally and then projecting the data onto the y-axis (sum over image rows). In the resulting 1D data array, fit the line positions for all orders (one fit per set of central peak and the two neighboring maxima) using an appropriate model function. This yields the line positions in pixels. Justify your choice of lineshape (e.g., Gaussian, Lorentzian, Voigt). Is the uncertainty given by the fit reliable? If not, estimate an appropriate error based on the shape and width of the fitted peak.

- For using the raw images in any image processing or coding software (e.g. ImageJ or python), the following parameters are important:
  - 16-bit unsigned image type
  - width = 2048 pixels, height = 512 pixels
  - little-endian byte order
  - higher wavelength values appear at the left hand side of the raw data image (spectrometer grating and camera mirror deflect higher wavelengths to the left hand side, so the camera image is displayed upside down in the spectrometer software, to get an increasing wavelength-axis to the right hand side)

- For each of the four magnetic field strengths, **plot the interference order k against the line position a of the π lines**. Fit a suitable polynomial function $k = f(a)$ (decide on the polynomial order based on the data). Since the spacing between successive orders corresponds to $f(\Delta a) = f(a_2) - f(a_1) = 1$, **determine the**

ratio $\delta a/\Delta a$ from your fit. Then compute $\delta\lambda$ using Eqn. 24, with the following parameters for the Lummer-Gehrcke plate: $n = 1.4567$, $d = 4.04\,\text{mm}$. Estimate the uncertainty of $\delta\lambda$ from the standard deviation of the order fit. For the wavelength λ, use the value you determined in Part II of the experiment.

- **Compute the Bohr magneton** μ_B for each of the four magnetic field values. Calculate the average and compare it with the literature value:

$$\mu_B = 9.274\,009\,49 \times 10^{-24}\,\text{J/T}.$$

- Optional (short/long evaluation): Plot the energy splitting ΔE versus the magnetic field B, fit a suitable model, and determine μ_B from the slope. Compare your result with the known literature value.

- **Compare the spectral resolution of the Czerny-Turner spectrometer with that of the Lummer-Gehrcke plate.**
  Hint: Reduce the slit width of the spectrometer and examine the tilted pattern of a single set of σ and π lines. Compare the horizontal distances to the vertical ones, caused by the Lummer-Gehrcke plate.

Part II: Precision Spectroscopy

Up to now, we have used the Lummer-Gehrcke plate to precisely determine small differences between wavelengths. However, the absolute wavelength was only determined approximately, due to the error-prone calibration of the Cerny-Turner spectrometer based on the grating rotation encoder. In the second part of the experiment, the absolute wavelength will be determined more precisely using known spectral lines of neon.

Calibration

Neon spectral lamp

In order to calibrate the spectrometer, for example to find the dispersion, reference lines will be necessary. For this reason, a neon spectral lamp will be used for calibration (Thorlabs CSL1). The necessary reference lines are listed in Tab. 2 (data from Thorlabs CSL1 Product Raw Data, data match the values of spectral line database of NIST, National Institute for Standards and Technology, `http://www.nist.gov`) Reference values are also displayed in an overview spectrum of the neon lamp (Fig.11).

Table 2: Neon reference lines for calibration
(source: Thorlabs CSL1 Product Raw Data and http://www.nist.gov)

Species	λ (nm)	Normalized Intensity	Species	λ (nm)	Normalized Intensity
Ne I	585.24878	1	Ne I	638.29914	0.28786
Ne I	588.1895	0.10191	Ne I	640.2248	0.52018
Ne I	594.4834	0.10492	Ne I	650.65277	0.25702
Ne I	597.55343	0.03624	Ne I	653.28824	0.09763
Ne I	602.99968	0.0376	Ne I	659.89528	0.12778
Ne I	607.43376	0.15792	Ne I	667.82766	0.25463
Ne I	609.6163	0.20348	Ne I	671.7043	0.16292
Ne I	614.30627	0.34517	Ne I	692.94672	0.19251
Ne I	616.35937	0.11925	Ne I	703.24128	0.5563
Ne I	621.72812	0.05824	Ne I	717.3938	0.02255
Ne I	626.64952	0.2073	Ne I	724.51665	0.19111
Ne I	630.47893	0.05761	Ne I	743.88981	0.04466
Ne I	633.44276	0.15745	Ne I	748.88712	0.03884

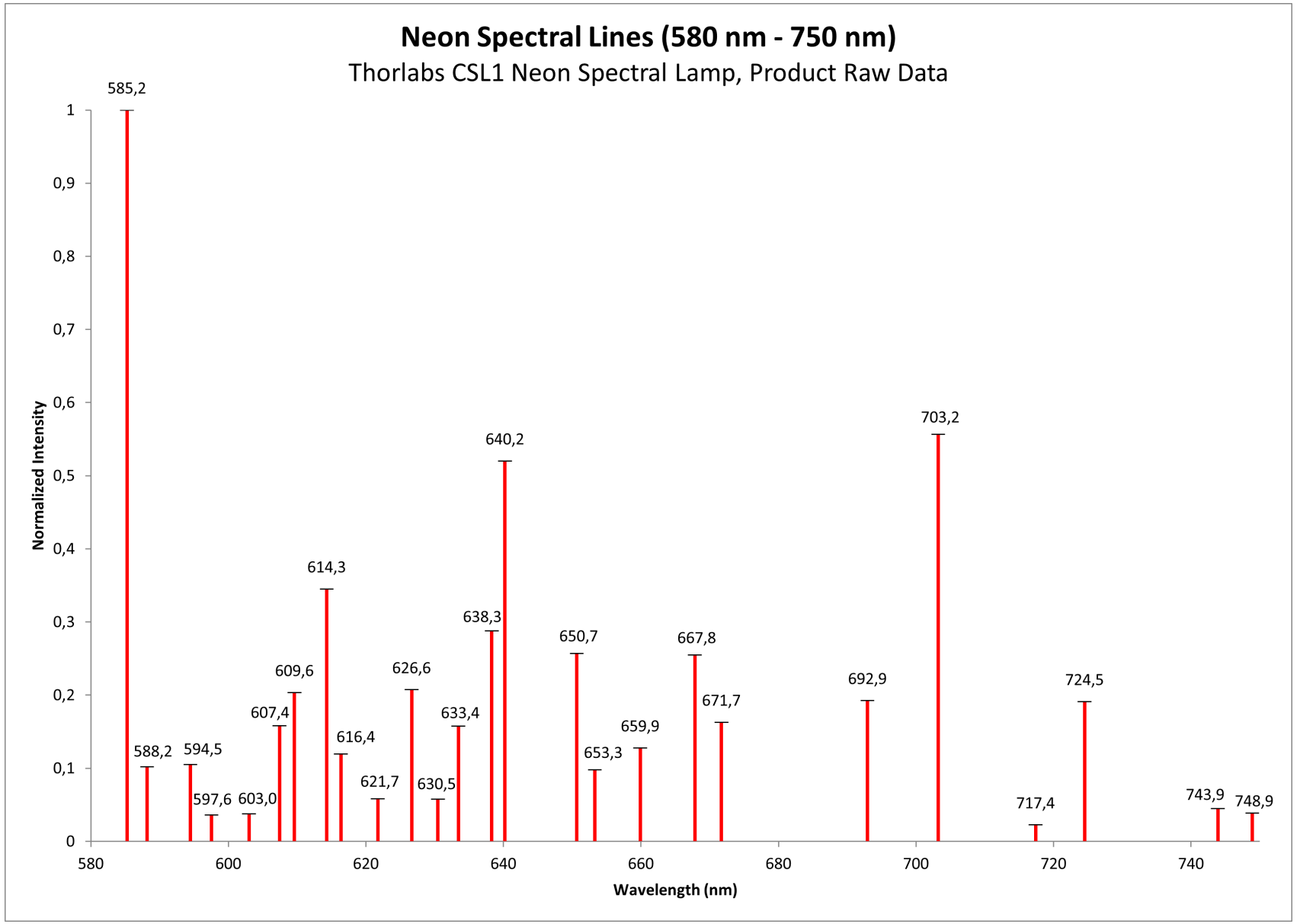

Figure 11: Normalized spectrum of the neon spectral lamp. The spectral lines correspond to the ones in Tab. 2. The wavelength is given in nanometers.

Part II: Execution

The aim of this experiment is to measure the wavelength of the red spectral line of Cadmium (Cd). Furthermore, another very weak line will be emitted from the Cd lamp with a wavelength of $\approx 652\,\text{nm}$. This is a spectral line of Xenon. Why can you observe this line? Which line do you expect? (To identify the line go to the NIST Atomic Spectra Database (`http://physics.nist.gov/PhysRefData/ASD/lines_form.html`))

- The bandwidth, which will be observed by the detector in one capture is $\approx 15\,\text{nm}$. Therefore, the grating must be rotated to map various regions of the Ne- and Cd-spectra. The grating converts the wavelength/energy into a geometrical value, namely the dispersion of the CCD sensor. Hence, you must proceed carefully with the alignment of the various components along the light path of the experimental setup.
- **Questions to be prepared:**
  - Are there sources of error in the setup?
  - How can these be prevented?
  - What line shape do the spectral lines have? Give reasons.
- Consider a **strategy for measuring the Cd-line** that will minimize the systematic errors (integration time of the camera, entrance slit width, calibration procedure etc.).
- Discuss this strategy with the supervisor before you start taking measurements.
- Use the provided optical fiber to guide the neon light to the spectrometer.
- Use the program **ReadOutUI** to take a spectrum of the neon spectral lamp and compare the image to the overview spectrum Fig.11, which is also displayed at the experiment. *Note:* Make sure you understood how the wavelength axis is orientated in the software display and in the raw data.
  Now find the characteristic region of the **neon spectrum** (around the expected wavelength for the Cadmium line) and make a separate recording of this region **(4 lines)**.
- Take an **image of the Cd-line** without moving the grating. The aquisition time and colour contrast must be adapted to the intensity of the Cd-lamp.

- Optional (short/long evaluation): Create an image, in which light from the Ne- and the Cd- lamps enter the spectrometer at the same time.

- Optional (short/long evaluation): Create an image of the whole neon spectrum by scanning the grating angle and taking overlapping images. Make sure that all images contain at least one line in the overlapping region in order to stitch all images together during evaluation.
  Note: The scanning range is limited by the distance between the peaks. Finish your scanning, if the distance between lines is too large to create overlapping images.

Part II: Evaluation

- **Fit the position of the 4 neon lines. Fit a calibration function**, such that pixels can be converted into wavelengths.

- **Determine the wavelength of the Cd-line.**

- Optional (short/long evaluation): Create a graph from an image in which both neon lines and Cd lines can be seen. Consider how to present the graph, such that the strong and weak intensity lines are portrayed equally.

- Optional (short/long evaluation): Create a graph of the whole neon spectrum, by stitching the images together.
  Compare yours with a graph plotted from the neon values of the Thorlabs Product Raw Data / NIST database (Tab. 2 or Download Thorlabs CSL1 Product Raw Data).

Preperation

Topics to be prepared are:

- Zeeman effect
- Hysteresis
- Hall effect
- Polarisation (filter, waveplate)
- Lummer-Gehrcke plate
- Grating spectrometer (Czerny-Turner Spectrometer)
- Camera sensors (CMOS vs. CCD)
- Internal photoelectric effect
- Line width and line broadening
- **Don't forget to bring an USB stick!**
  The experiment PC doesn't have network connection.

Appendix

A1 Operating The SPEX-Spectrometer

- **Check if the camera is turned on:** The power supply is located at the rear foot of the stand, mounted to the aluminum frame. *Note:* The spectrometer software will show an error message if no camera is available.

- **Prepare the data folder:** Open the folder `F44_Zeeman-Effekt_Daten` on the desktop (`C:\Users\praktikum\Desktop\F44_ZeemanEffekt_Daten`) and create a subfolder named with the current date and your team name.

- **Open the Qt Creator software:** Launch `Qt Creator` from the desktop or from `C:\Qt\Tools\QtCreator\bin\qtcreator.exe`.

- **Open the project "ReadOutUI":** Either open it from the "Projects" section or via the menu: `File` → `Open File or Project (Ctrl+O)` → `C:\Users\praktikum\Documents\QtTests\ReadOutUI\ReadOutUI.pro`

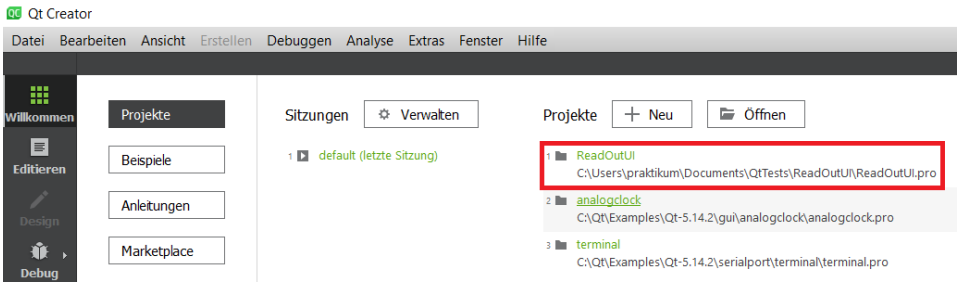

Figure 12: Open project ReadOutUI

- **Run the software:** Click the green play button ("Run") in the lower left. The `MainWindow` opens (Fig. 13).

- **Connect to the spectrometer:** Click the "Connect" button in the MainWindow (Fig. 14). The software connects to the stepper motor of the grating spectrometer. (Button changes to "Disconnect".)

- **Check grating position:** Check if current wavelength of MainWindow matches with wavelength of spectrometer dial (in Å). If not: type in the dial value into the text field on the right hand side of "Dial [Ångström]" and press "Recalibrate to Dial" button.

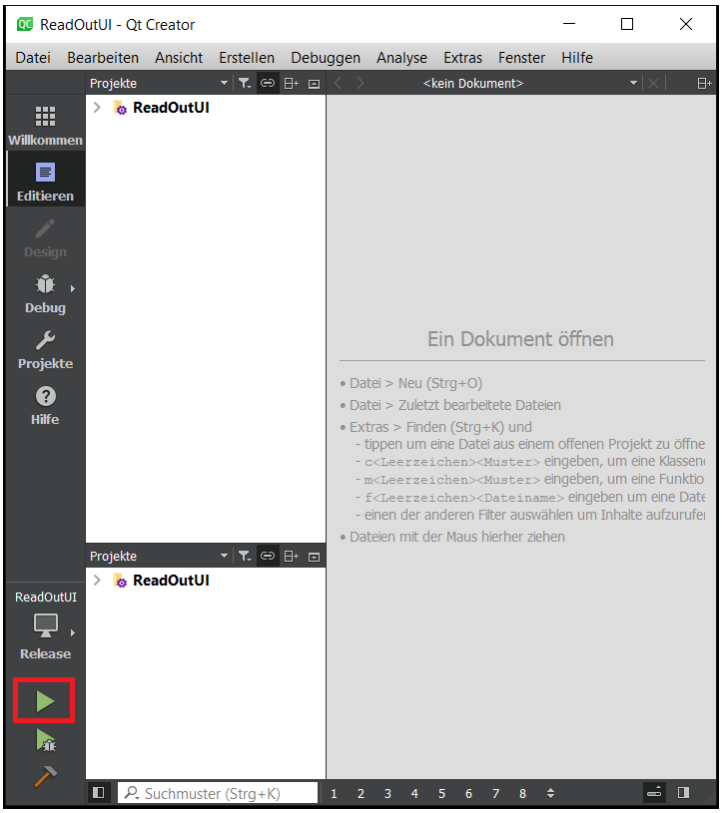

Figure 13: Execute project ReadOutUI

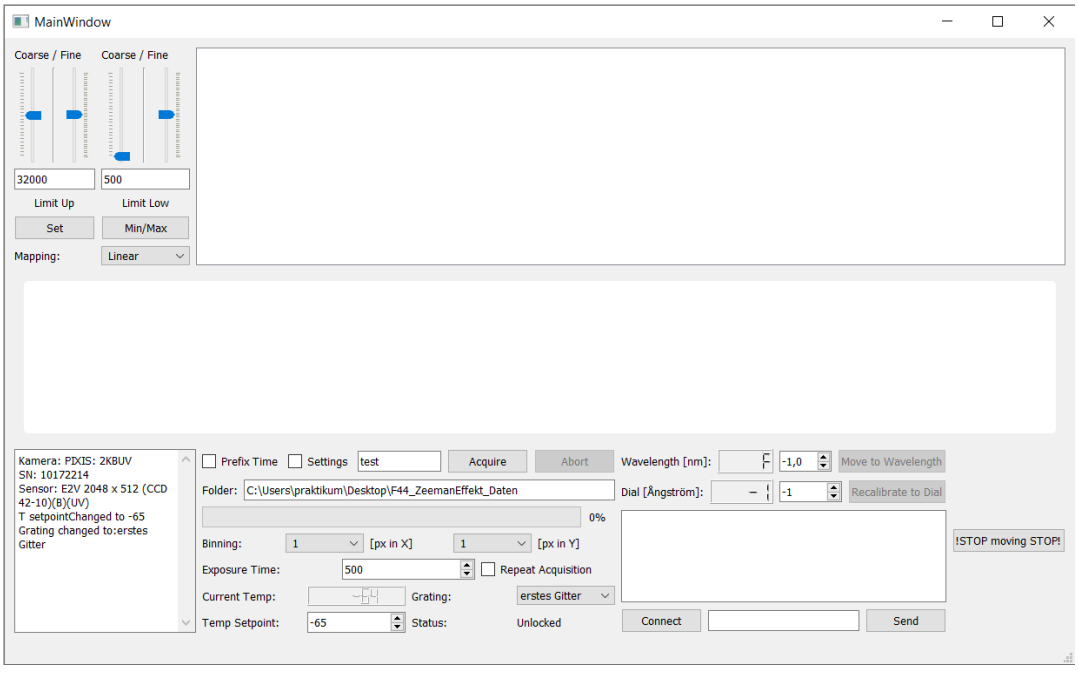

Figure 14: Graphical user interface "MainWindow" of ReadOutUI

- **Move spectrometer to red Cd line:** Enter 639 in the “Wavelength [nm]” field and press “Move to Wavelength”.
- **Set correct data path:** In the “Folder” input field, append your own subfolder name. *Caution:* If the subfolder does not exist or is entered incorrectly, the spectrometer software will not create it and no data will be saved!
- **Perform a test measurement:** Make sure the checkbox “Prefix Time” is **unchecked**. Click the “Acquire” button (set exposure time to 500 ms, switch off room light).

  If necessary
  - adjust the grating further to find and center the line in the camera image
  - reduce or increse the exposure time (e.g., to 300 ms)
  - optimize the contrast in the image display using the “Min/Max” button or the sliders located to the left of the camera image
- If you don’t see any interference pattern or even any light signal on the camera image, call your tutor for help. The beam path may need to be realigned (if not hitting the entrance slit or first spectrometer mirror).
- **Optimizing image quality:** Once you see the interference pattern on the camera image, activate checkbox "Repeated Aquisition" to start continous aquisition (The raw file will now be overwritten with each new exposure).
  - Vary the position of lens 3 along the beam path to check if a better image sharpness can be reached.
  - Carefully turn the knobs of mirror 2 to vary the position of the interference pattern within the image to get approx. five well seperated lines in the field of view.
  - The intensity should be maximized, and the slit should be illuminated as uniformly as possible — aim for a plateau in the intensity profile (Fig. 15).

  Stop aquisition and uncheck "Repeated Aquisition".
- **Recording of required experiment data:** For every measurement enter a meaningful filename in the `MainWindow` of the Qt Creator software. You also may check "Prefix Time" to add a time stamp to the file name, to be sure no raw file will

be overwritten. Check your subfolder after first measurement to verify that a camera image (raw file) has been saved.

- **Measurement of neon spectral lines:** Start with open entrance slit and move and tilt the end of the fiber in front of the slit until you can see any kind of light signal. Close the slit size step by step and refine the position of fiber each step. When the slit becomes very narrow the line structure of the neon light is revealed.

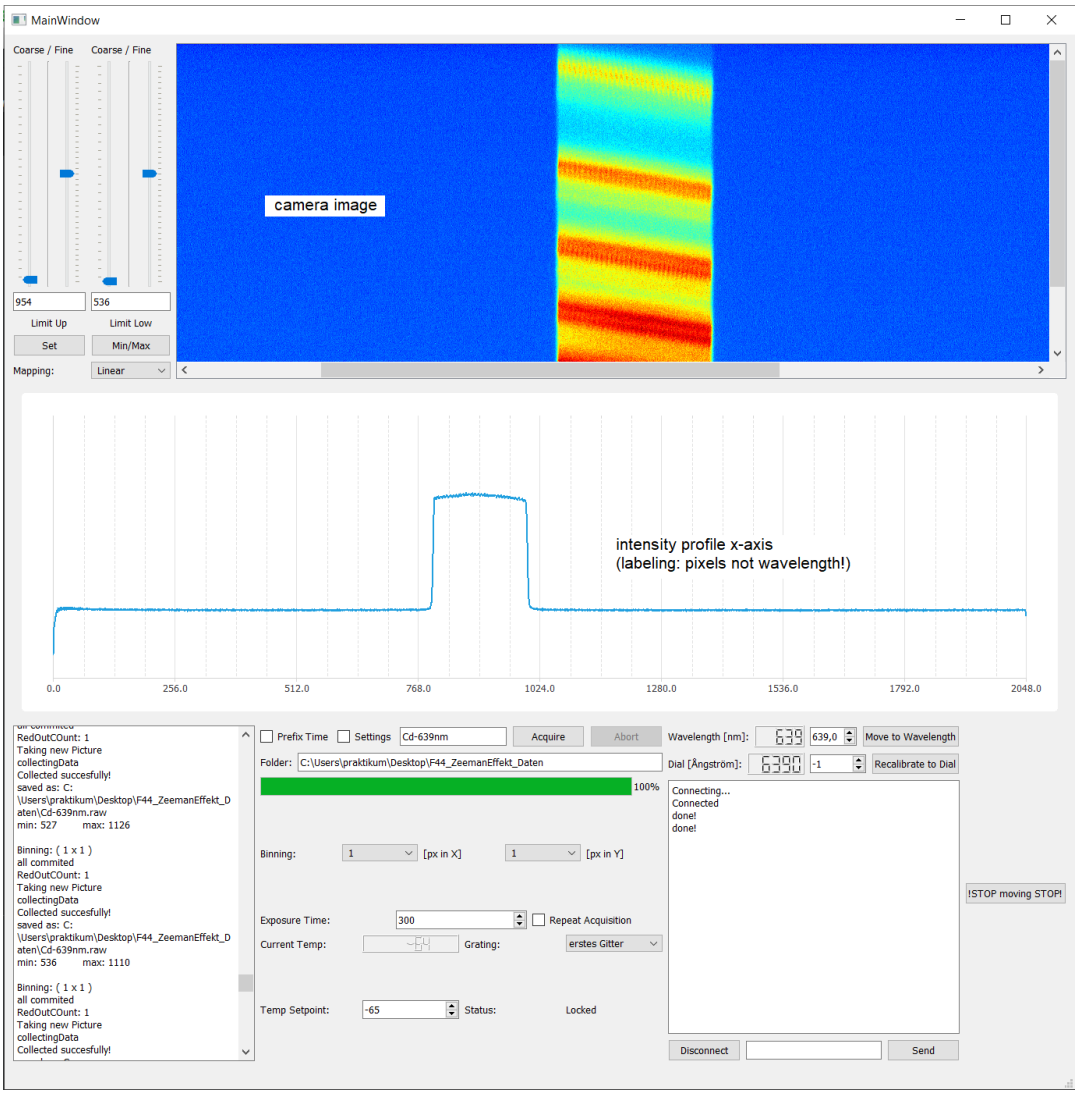

Figure 15: Interference pattern of red Cd line

A2 Realignment Of Beam Path Into Spectrometer

If beam path is misaligned and does not hit the entrance slit.

- **Move spectrometer to zeroth diffraction order:** Enter 5 in the "Wavelength [nm]" field and press "Move to Wavelength".
  Warning: Do not enter values less than 5 because the stepper motor could run into a hardware limit and get stuck there.
  Warning: This movement takes about 20 minutes and cannot be interrupted with the button "!STOP moving STOP!" (currently non-functional). To stop movement enter a new wavelength into the text field that is near to the current dial value and press again "Move to Wavelength".

- **Transversal magnet configuration:** If necessary rotate electromagnet to transversal position and place the three-sided aluminum cover between pole-pieces and Lummer-Gehrcke plate to block stray light. Place the Cd-lamp between the pole pieces and switch lamp on.

- **Insert lens 1 with red filter** into the holder.

- **Visualize intermediate image:** Mount ThorCam into the empty post holder behind the horizontal slit. The top of the camera housing should be at the same hight as the top edge of mirror 1. Place lens 2 150 mm in front of ThorCam. Start ThorCam software and position the interference pattern into the camera field of view by moving lens 2 carefully left/right and forwards/backwards. (If necessary remove the horizontal slit for this step and remount it after having found the sharp and well separated interference lines in the camera image.)

- **Fix lens 2 at this position,** and remove ThorCam out of the post holder. (Don't move the post holder itself!)

- **Insert lens 1 with transparent base plate** into the holder.

- **Align the optical path:** If neccessary: align the optics (M1, L3, M2 and M3) like it is shown in Fig. 10 and position lens 3 as calculated with Eqn. 37.

- **Align light spot onto entrance slit:** Use the turning knobs of the kinematic mirror mounts to direct the beam onto the entrance slit. Try to pass L3 in the middle of the lens.

Important: Use a white card, switch off the room lights and rotate the monitor away to track the path of the light spot.

- **Take a test image,** to check if you already can see some light signal on the detector. (Again: Turn off the room lights and rotate the monitor away — stray light also reaches the camera in the zeroth diffraction order. Optimize image contrast by clicking on "Min/Max" button.)
- **If you don't see any light signal,** you most probably enter the spectrometer under a wrong angle - not hitting the first internal curved mirror:
  - Remove the cover of the spectrometer and follow the light spot inside the spectrometer. It needs to be very dark inside the room and no other light shall hit your eyes to be able to see the light spot inside the spectrometer - it is very weak.
  - Use M2 and maybe M3 (less effect than M2) to pass through the entrance slit and hit the first curved mirror.
    If you managed to hit the first spectrometer mirror you should be able to see a light signal on the camera!
  - Close the spectrometer and perform another test measurement (Min/Max button!)
- **Once you see the interference pattern** in the camera, switch to continous acquisition and optimize the position of the stripe pattern in the camera field of view. You may use M2 and also the hight and position (along the beam path) of L3 for that.
  - The pattern should appear in a region where the stripes are as far apart as possible.
  - The intensity should be maximized, and the slit should be illuminated as uniformly as possible — aim for a plateau in the intensity profile.
- **Move spectrometer to first diffraction order** of the red Cd line: enter 639 nm and move to this wavelength using the "Move to wavelength" button (this again takes about 20 minutes; note that the middle of the camera image shows a slight wavelength offset compared to the spectrometer dial).
- **Check again the interference pattern image** and optimize again position, sharpness and contrast of the stripe pattern

A3 Operating Thorlabs Zelux Camera

Only if you need to display the intermediate image of the interference pattern.

ThorCam software

- Open ThorCam software from desktop
- Click on camera symbol and select camera (CS165CU), (Fig. 16)

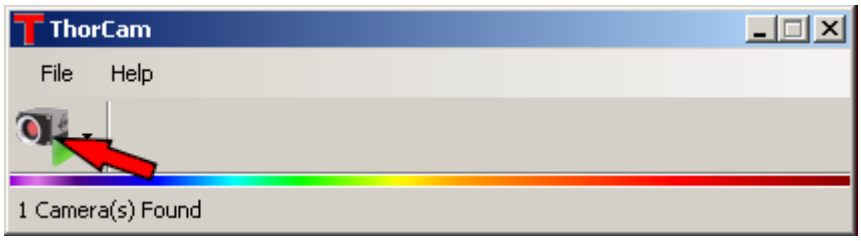

Figure 16: Starting screen of ThorCam

- Now the program opens. When you click on start capture, the camera image is shown on the screen.
  To optimize your image (e.g. so you can see faint lines and no lines are saturated), you can change the options. After you acquired an image to your satisfaction, you can save the image (Fig. 17).

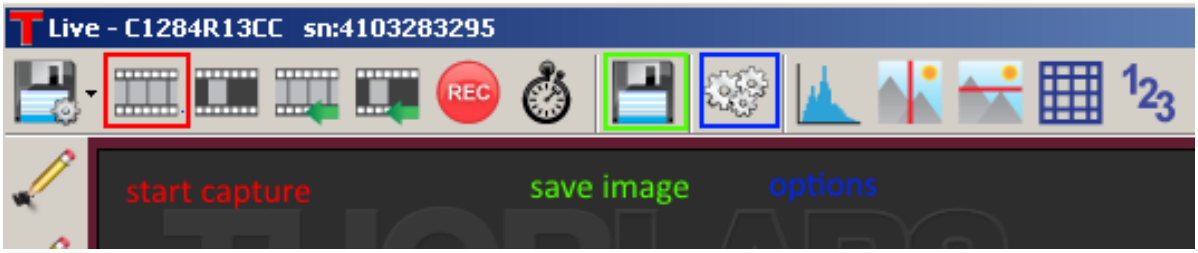

Figure 17: ThorCam toolbar

- In the Options section you can change the gain ('Image'-register) and the exposure time ('Camera'-register) to improve your image (Fig. 18). You can also reduce the color gain of blue and green and increase the red one.

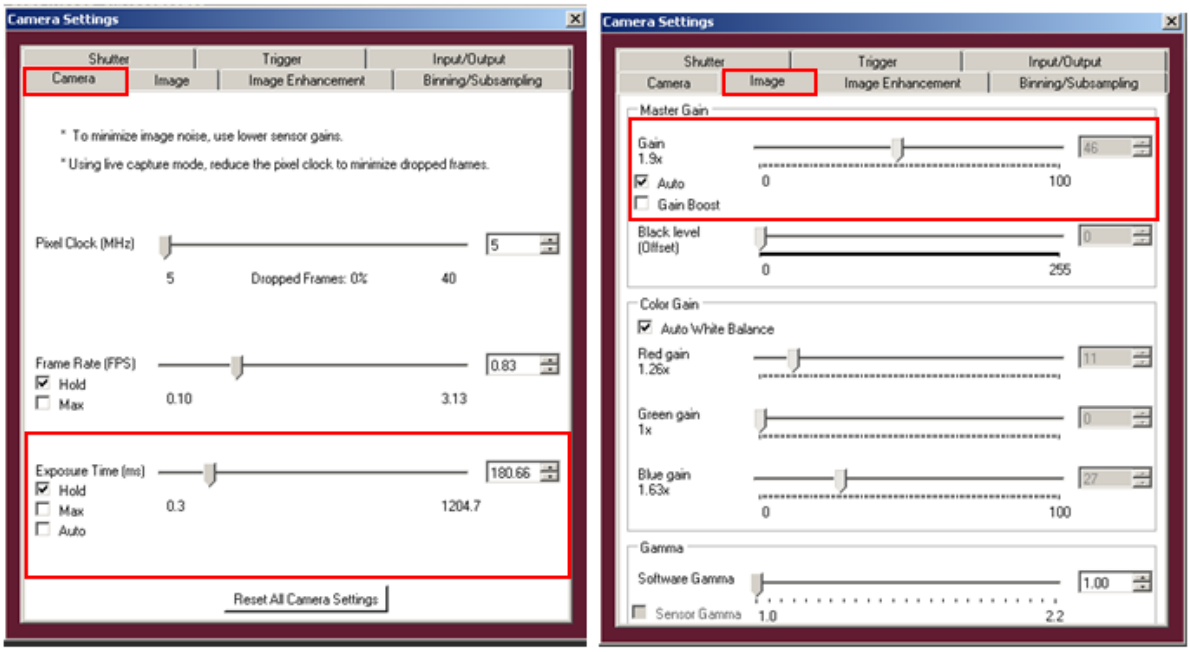

Figure 18: ThorCam option settings