Lecture Note 1

Quantum Optics and Quantum Information

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Outlines

- A quick review of quantum mechanics
- Quantum superposition and noncloning theorem
- Quantum Zeno effect
- Quantum entanglement
- Quantum nonlocality

Basic Principles of Quantum Mechanics

- The state of a quantum mechanical system is completely specified by wave function $\psi(x,t)$, which has an important property that $\psi(x,t)\psi^*(x,t)dx^3$ is the probability that the particle lies in the volume element dx^3 at time t, satisfying the normalized condition

$$\int_{-\infty}^{\infty} \psi(x,t)\psi^*(x,t)dx^3 = 1$$

- To every observable in classical mechanics there corresponds a linear, Hermitian operator in quantum mechanics (indicates superposition principle).

Basic Principles of Quantum Mechanics

- In any measurement of the observable associated with operator $\hat{A}$, the only values that will ever be observed are the eigenvalues a_i , which satisfy the eigenvalue equation

$$\hat{A}\psi_i = a_i\psi_i$$

If a system is in a state described by a normalized wave function ψ, then the average value of the observable corresponding to

$$<\hat{A}> = \int_{-\infty}^{\infty} \psi^* \hat{A}\psi dx^3$$

Basic Principles of Quantum Mechanics

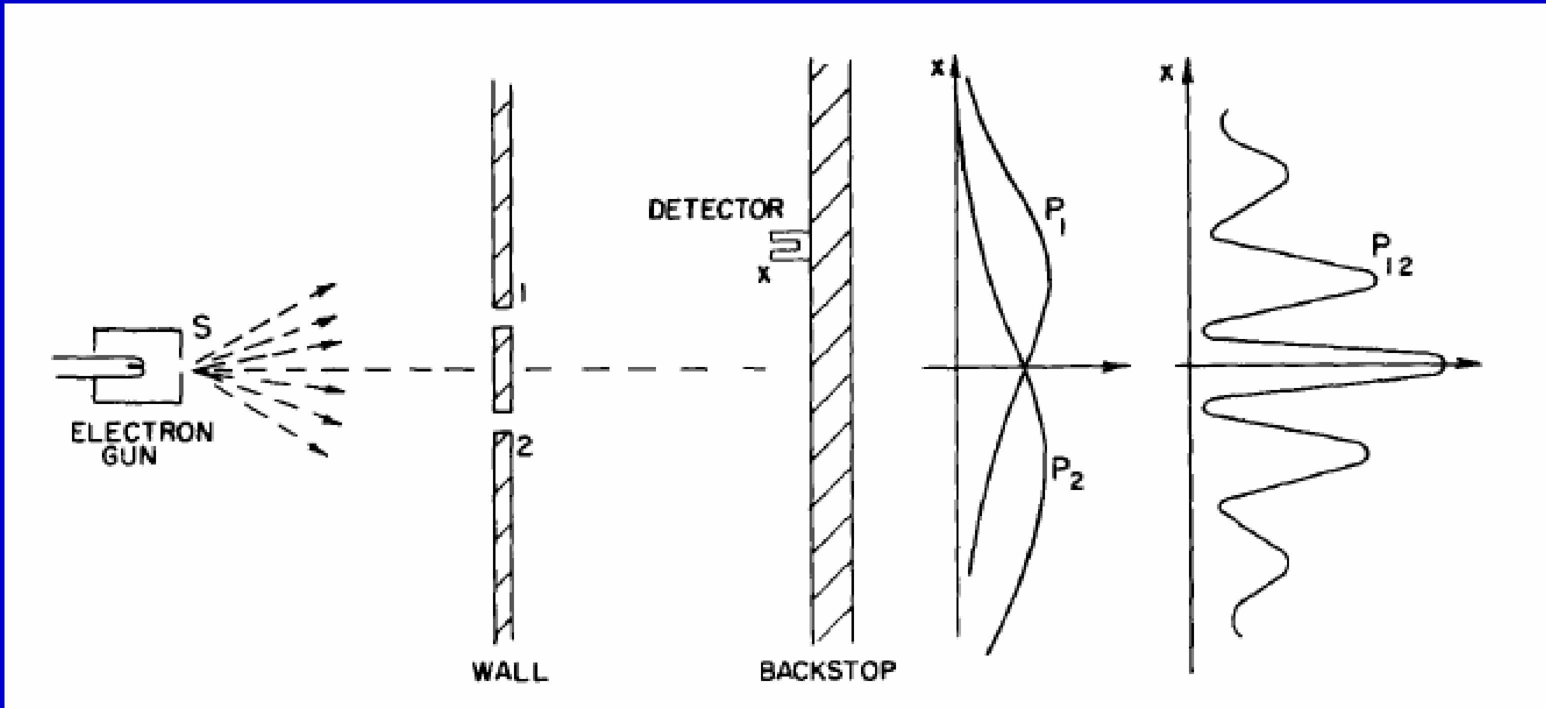
- The wave function or state function of a system evolves in time according to the time-dependent Schrödinger equation

$$i\eta\frac{\partial\psi}{\partial t} = H\psi$$

where H is the Hamiltonian.

- The total wave function must be antisymmetric (or symmetric) with respect to the interchange of all coordinates of one fermion (boson) with those of another.

Quantum Superposition Principle



$$p_{12} = |\phi|^2,$$

$$\phi = \phi_1 + \phi_2,$$

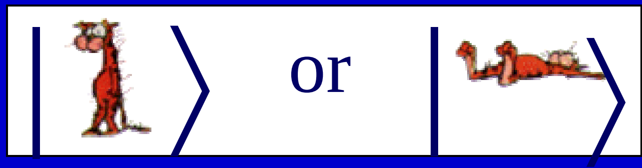
$$p_1 = |\phi_1|^2, \; p_2 = |\phi_2|^2.$$

Quantum Superposition Principle

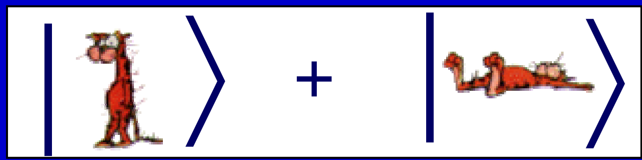
which slit?

Quantum Superposition Principle

Classical Physics:
"bit"



Quantum Physics:
"qubit"



Quantum foundations: Bell's inequality, quantum nonlocality…
Quantum information processing: quantum communication, quantum computation, quantum simulation, and high precision measurement etc …

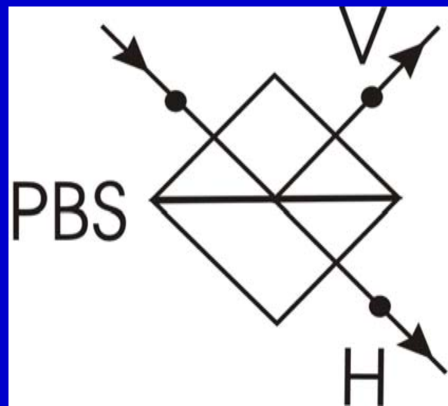
Qubits: Polarization of Single Photons

One bit of information per photon
(encoded in polarization)

$|H>=|"0">$
$|V>=|"1">$

Qubit: $$|\Phi\rangle = \alpha|H\rangle + \beta|V\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1$$



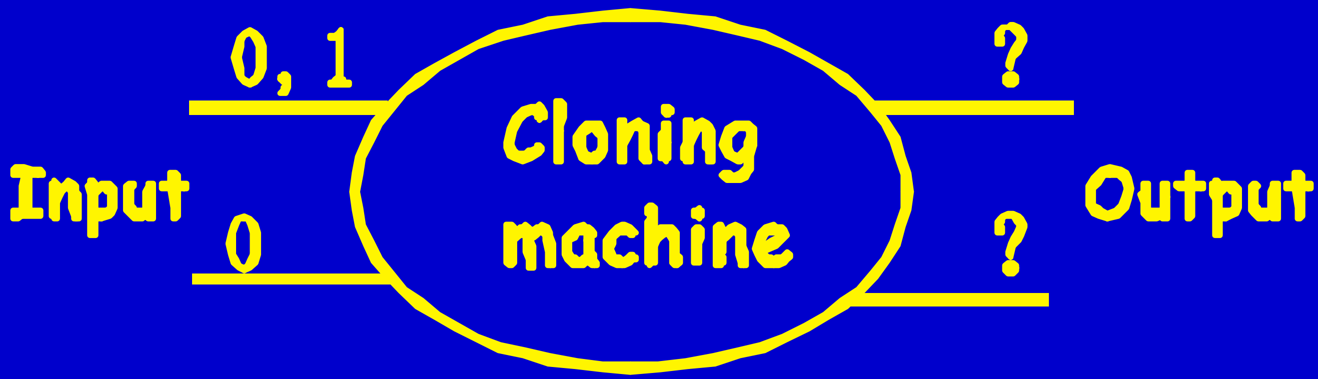
$|\beta|^2$

$|\alpha|^2$

Non-cloning theorem:

An unknown quantum state can not be copied precisely!

Non-Cloning Theorem



$$00 \rightarrow 00,$$

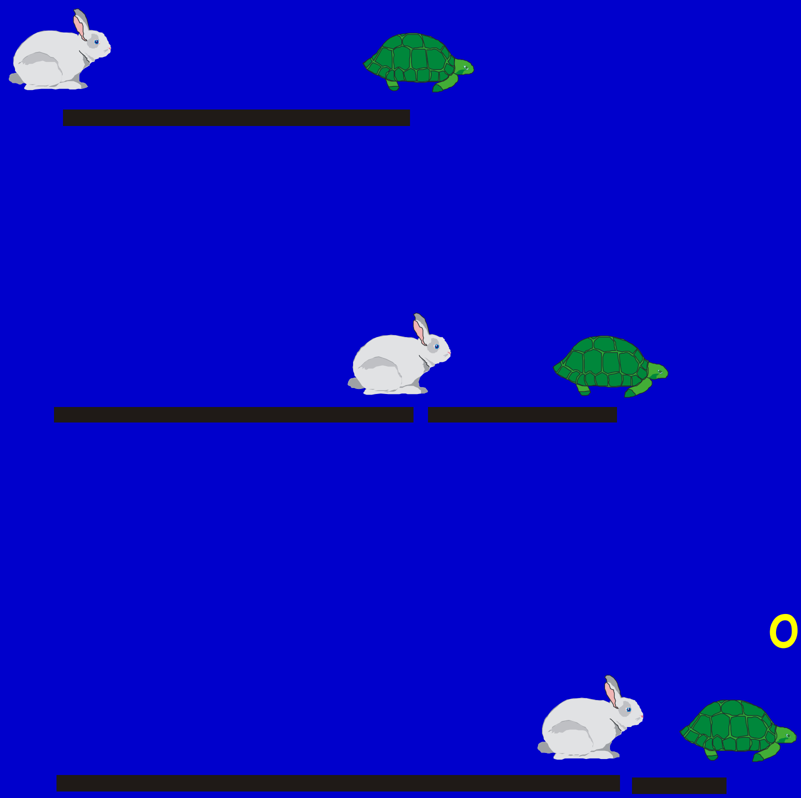
$$10 \rightarrow 11,$$

According to linear superposition principle

$$(0+1)0 \rightarrow 00+11 \neq (0+1)(0+1)$$

W. Wootters & W. Zurek, Nature 299, 802 (1982).

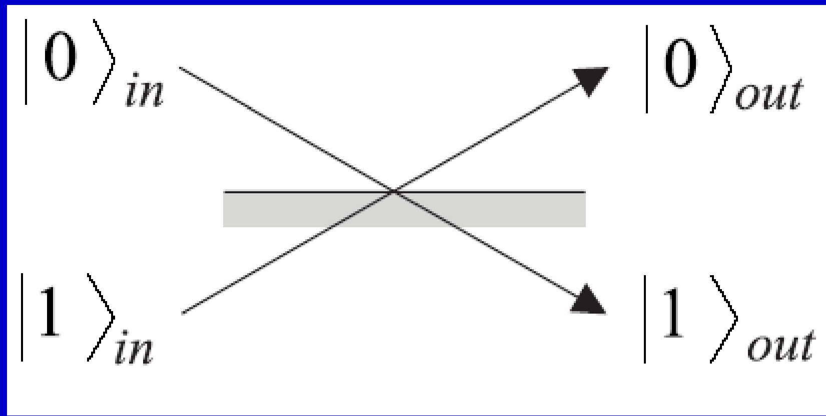
Zeno Paradox

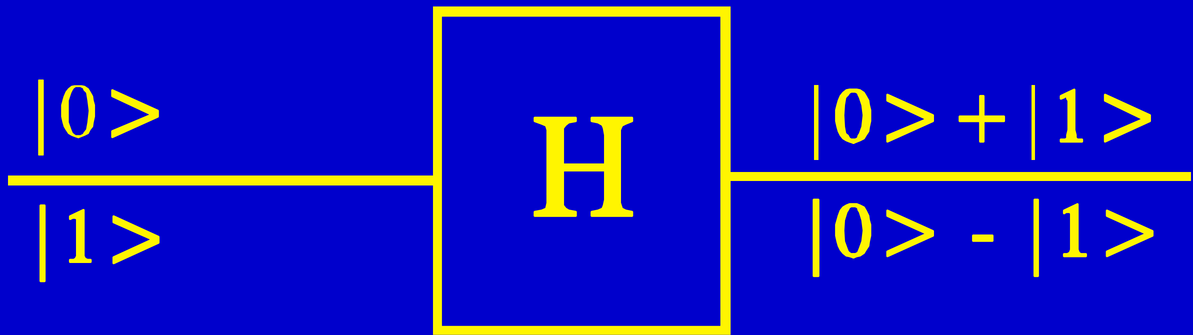


Origin of Zeno effect

Can the rabbit overtake the turtle?

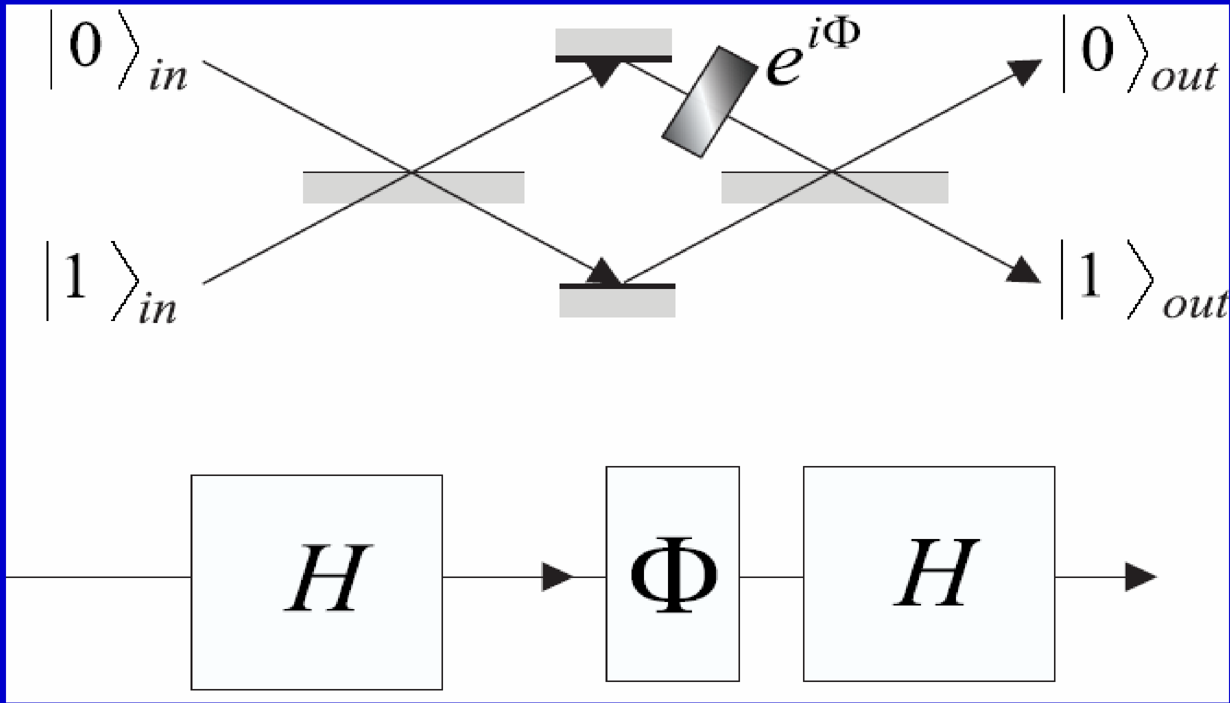
Quantum Zeno Effect





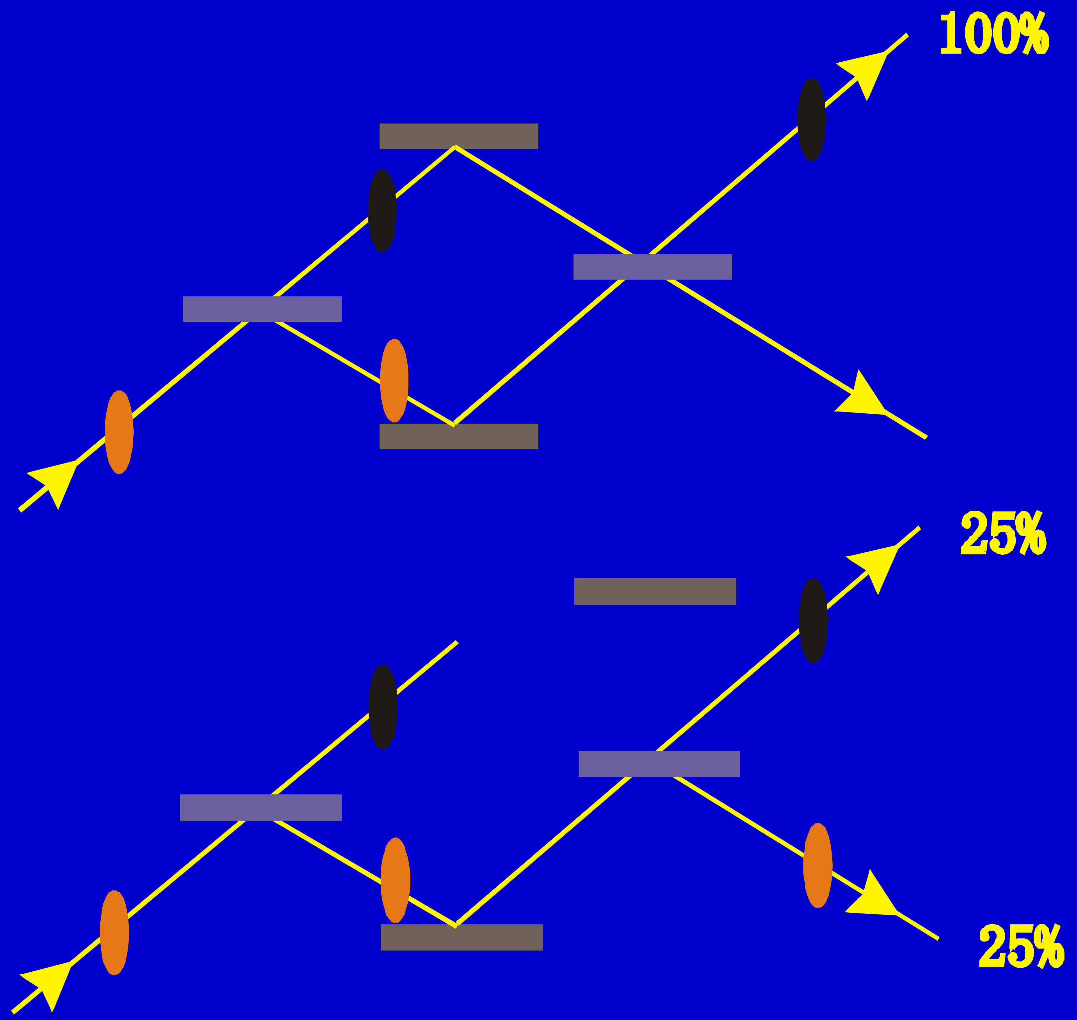
The Hadamard gate

Quantum Zeno Effect



$$H\Phi H|0\rangle = \frac{1}{2}((e^{i\varphi}+1)|0\rangle + (e^{i\varphi}-1)|1\rangle)$$

Quantum Zeno Effect



Interaction-free measurement !

Quantum Zeno Effect

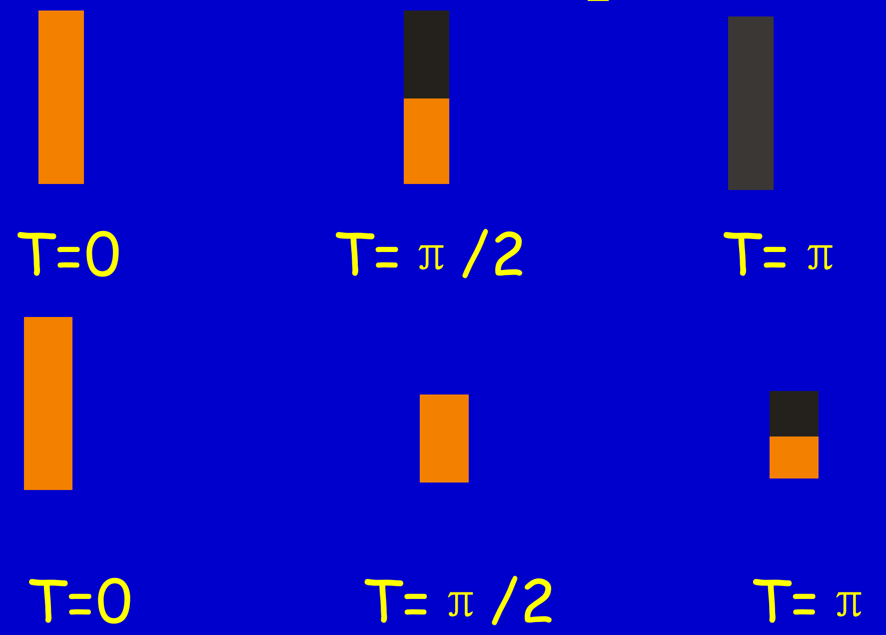
Considering neutron spin evolving in magnetic field, the probability to find it still in spin up state after time T is

$$p = \cos^2\left(\frac{\omega T}{2}\right)$$

where ω is the Larmor frequency .

Quantum Zeno Effect

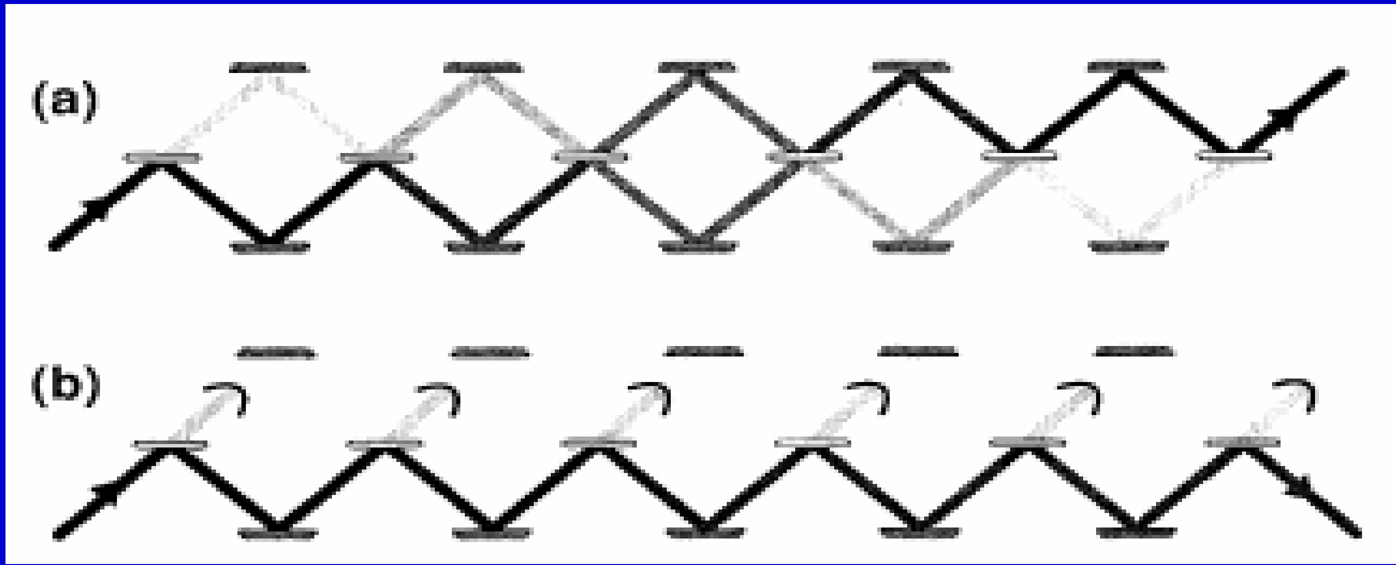
$$G\ (\text{cake is good}) = G_0 \times \frac{1+\cos T}{2}, (\omega = 1)$$



If we cut the bad part of the cake at time $T = \pi/2$, then at $T = \pi$ we have

$$G = 1/4 \times G0$$

Experiment



$$P = [\cos^2(\frac{\pi}{2N})]^N$$

In the limit of large N

$$P = 1 - \frac{\pi^2}{4N} + O(N^{-2})$$

P. Kwiat et. al., Phys. Rev. Lett. 74 4763 (1995)

Polarization Entangled Photon Pair

Bell states - maximally entangled states:

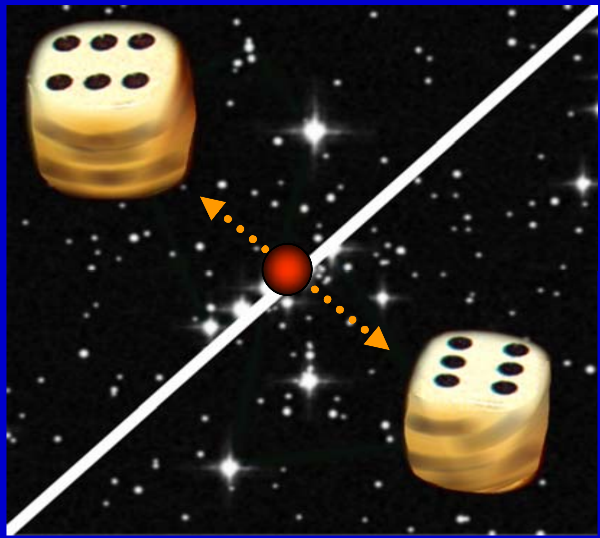
$$|\Phi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |H\rangle_2 \pm |V\rangle_1 |V\rangle_2\right)$$

$$|\Psi^{\pm}\rangle_{12} = \frac{1}{\sqrt{2}}\left(|H\rangle_1 |V\rangle_2 \pm |V\rangle_1 |H\rangle_2\right)$$

Singlet:

$$|\Psi^{-}\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2)$$

$$= \frac{1}{\sqrt{2}}(|H'\rangle_1 |V'\rangle_2 - |V'\rangle_1 |H'\rangle_2)$$



where

$$|H'\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$$

$$|V'\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$$

45-degree polarization

Polarization Entangled Photons

GHZ states
- three-photon maximally entangled states:

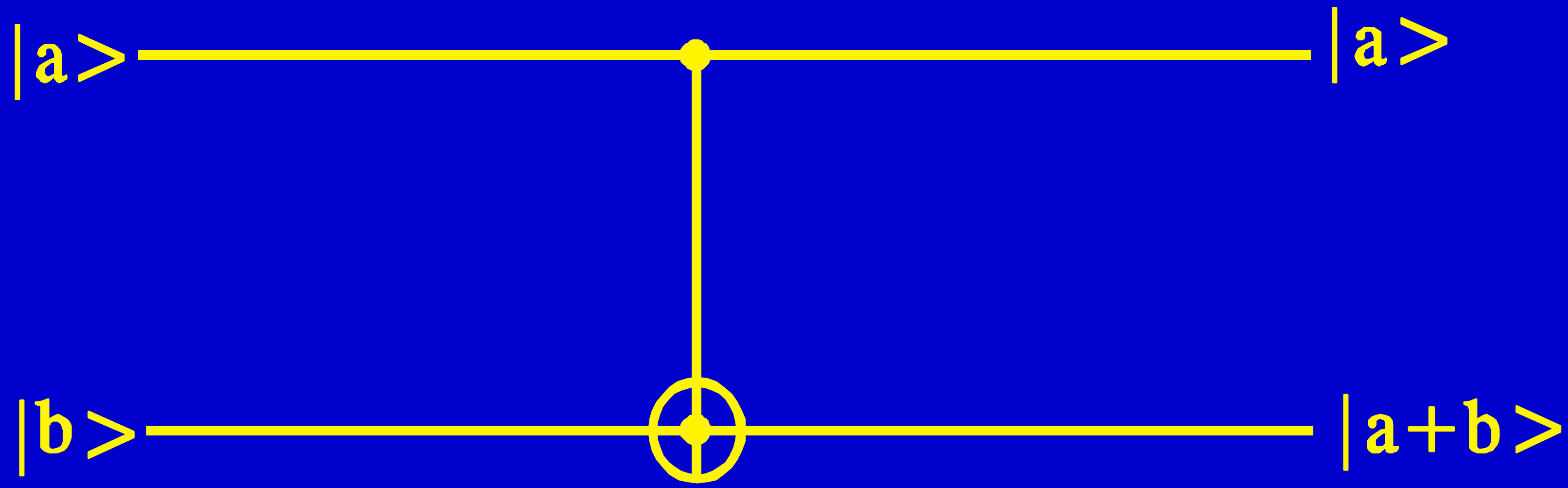
$$\left|\Phi^{\pm}\right\rangle_{123} = \frac{1}{\sqrt{2}}\left(\left|H\right\rangle_1 \left|H\right\rangle_2 \left|H\right\rangle_3 \pm \left|V\right\rangle_1 \left|V\right\rangle_2 \left|V\right\rangle_3\right)$$

$$\left|\Psi^{\pm}\right\rangle_{123} = \frac{1}{\sqrt{2}}\left(\left|H\right\rangle_1 \left|H\right\rangle_2 \left|V\right\rangle_3 \pm \left|V\right\rangle_1 \left|V\right\rangle_2 \left|H\right\rangle_3\right)$$

$$\left|\Xi^{\pm}\right\rangle_{123} = \frac{1}{\sqrt{2}}\left(\left|H\right\rangle_1 \left|V\right\rangle_2 \left|H\right\rangle_3 \pm \left|V\right\rangle_1 \left|H\right\rangle_2 \left|V\right\rangle_3\right)$$

$$\left|\Theta^{\pm}\right\rangle_{123} = \frac{1}{\sqrt{2}}\left(\left|H\right\rangle_1 \left|V\right\rangle_2 \left|V\right\rangle_3 \pm \left|V\right\rangle_1 \left|H\right\rangle_2 \left|H\right\rangle_3\right)$$

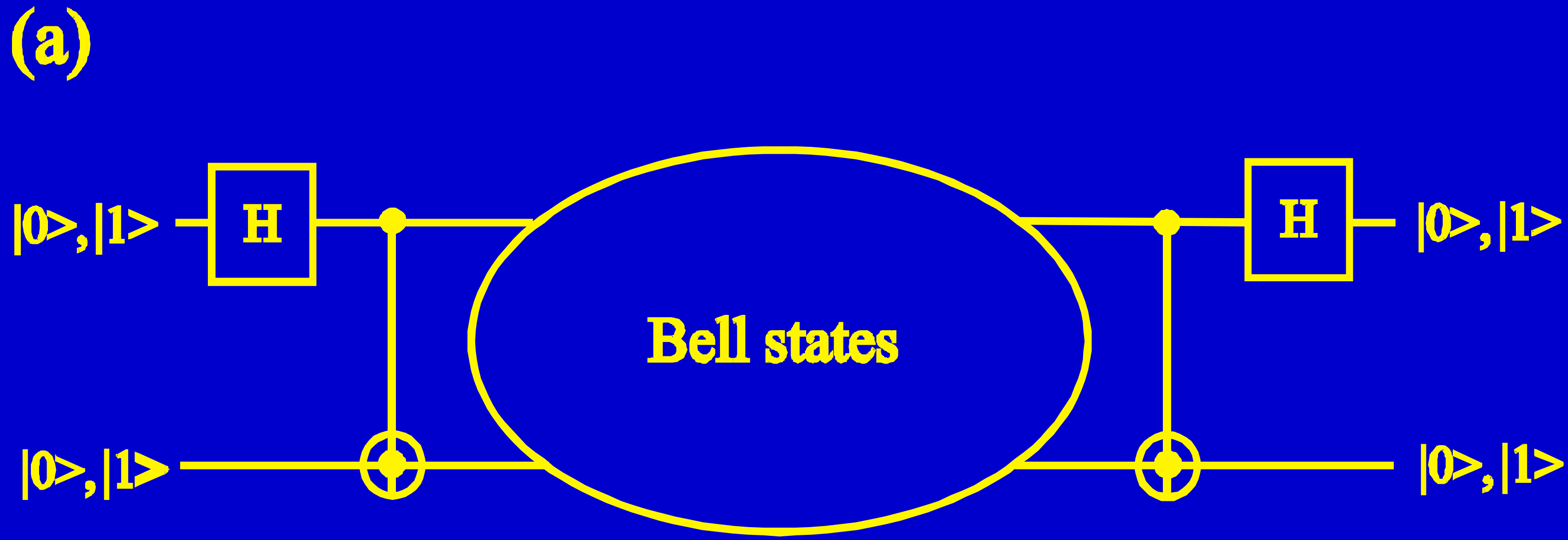
Manipulation of Entanglement



Quantum Controlled-NOT

$$00 \rightarrow 00$$
$$01 \rightarrow 01$$
$$10 \rightarrow 11$$
$$11 \rightarrow 10$$

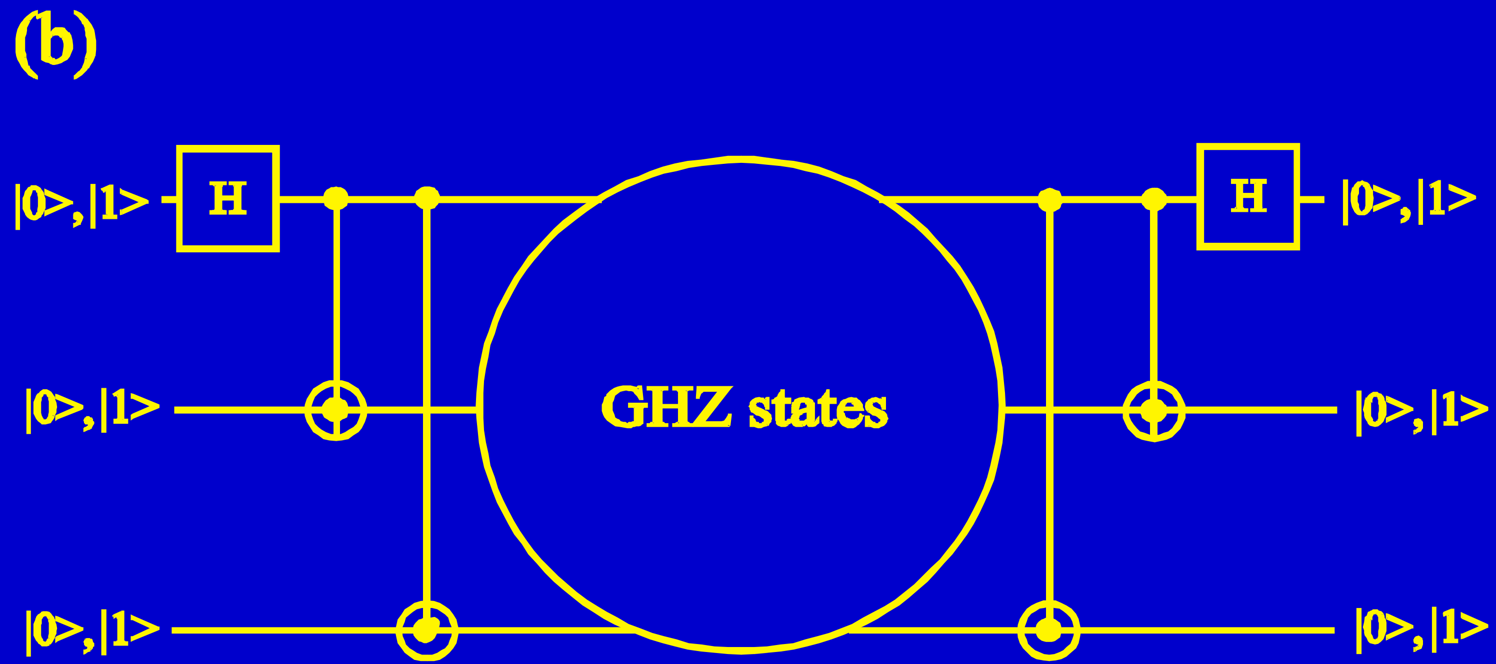
Manipulation of Entanglement



$$00 \to (0+1)0 = 00+10 \to$$

$$(00+11) \to (00+10) = (0+1)0 \to 00$$

Manipulation of Entanglement



$$000 \rightarrow (0+1)00 = (00+10)0 \rightarrow (00+11)0$$
$$= 000+110 \rightarrow 000+111 \rightarrow 000+110$$
$$= (00+11)0 \rightarrow (00+10)0 = (0+1)00 \rightarrow 000$$

Einstein-Poldosky-Rosen Elements of Reality

MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

Completeness

In a complete theory there is an element corresponding to each element of reality.

Locality

. . . *If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.*

non-commuting operators, i.e.,
momentum **P** and position **X** of a particle

Bohm's Argument

Bohm converted the original thought experiment into something closer to being experimentally testable.

Polarization Entangled Photon Pairs shared by Alice and Bob

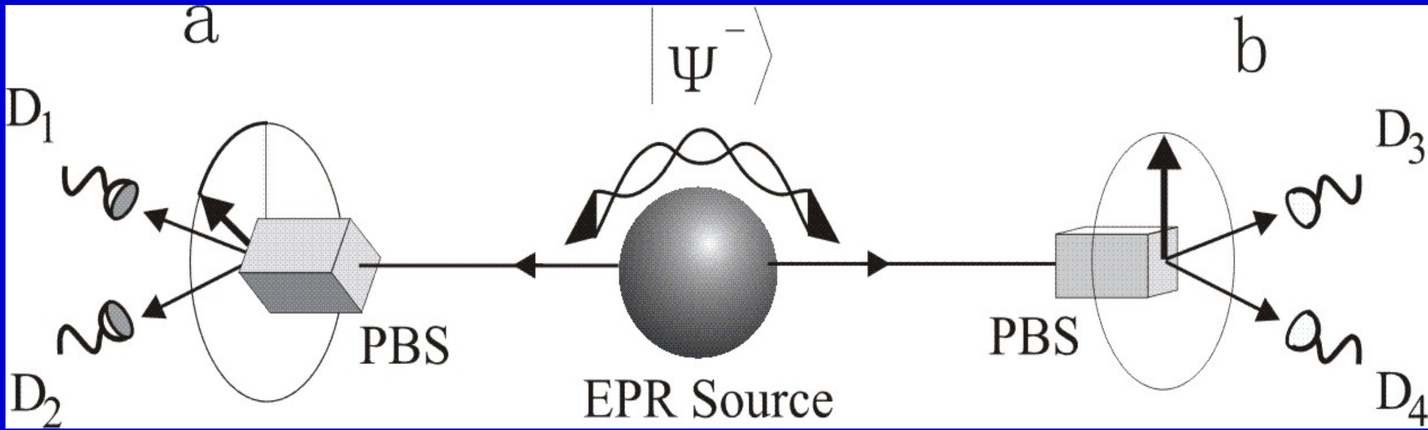
$$H = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad V = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \sigma_z$$

$$\left.\begin{aligned} H' &= \frac{1}{\sqrt{2}}(H+V) \\ V' &= \frac{1}{\sqrt{2}}(H-V) \end{aligned}\right\}, \quad \sigma_x$$

$$\left.\begin{aligned} R &= \frac{1}{\sqrt{2}}(H+iV) \\ L &= \frac{1}{\sqrt{2}}(H-iV) \end{aligned}\right\}, \quad \sigma_y$$

$$\begin{aligned} |\Psi^-\rangle_{12} &= \frac{1}{\sqrt{2}}(|H\rangle_1 |V\rangle_2 - |V\rangle_1 |H\rangle_2) \\ &= \frac{1}{\sqrt{2}}(|H'\rangle_1 |V'\rangle_2 - |V'\rangle_1 |H'\rangle_2) \\ &= \frac{1}{\sqrt{2}}(|R\rangle_1 |L\rangle_2 - |L\rangle_1 |R\rangle_2) \end{aligned}$$

Bohm's Argument



According to the EPR argument, there exist three elements of reality corresponding to σ_x, σ_y and σ_z !

However, quantum mechanically, σ_x, σ_y and σ_z are three noncommuting operators !

Einstein-Podolsky-Rosen Elements of Reality

In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality.

Quantum-mechanical description of physical reality cannot be considered complete!

Bell's Inequality and Violation of Local Realism

The limitation of the EPR !

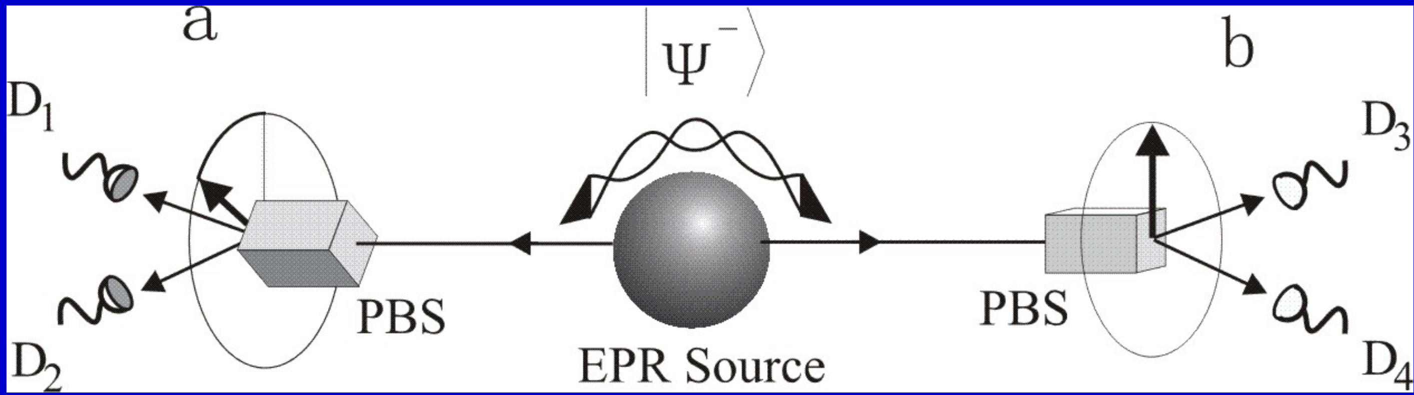
Both a local realistic (LR) picture and quantum mechanics (QM) can explain the perfect correlations observed.

Non-testable!

[J. S. Bell, Physics 1, 195 (1964)]

Bell's inequality states that certain statistical correlations predicted by QM for measurements on two-particle ensembles cannot be understood within a realistic picture based on local properties of each individual particle.

Sakurai's Bell Inequlaity



$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|H_1\rangle|V_2\rangle - |V_1\rangle|H_2\rangle)$$

Correlation measurements between Alice's and Bob's detection events for different choices for the bases (indicted by **a** and **b** for the orientation of the PBS).

Sakurai's Bell Inequlaity

Alice	Bob	Probability
a b c	a b c	
+ + +	− − −	P_1
+ + −	− − +	P_2
+ − +	− + −	P_3
+ − −	− + +	P_4
− + +	+ − −	P_5
− + −	+ − +	P_6
− − +	+ + −	P_7
− − −	+ + +	P_8

Pick three arbitrary directions **a**, **b**, and **c**

$$P(a+,b+) = P_3 + P_4$$

$$P(a+,c+) = P_2 + P_4$$

$$P(c+,b+) = P_3 + P_7$$

It is easy to see

$$P_3 + P_4 \leq P_3 + P_4 + P_2 + P_7$$

Sakurai's Bell Inequlaity

$$P(a+,b+) \le P(a+,c+) + P(c+,b+)$$

The quantum-mechanical prediction is

$$P(a+,b+) = \frac{1}{2}(\sin\frac{a-b}{2})^2$$

For example $a = 90^\circ$, $c = 45^\circ$, $b = 0^\circ$, the inequality would require

$$\frac{1}{2}\sin^2 45^\circ \le \frac{1}{2}\sin^2 22.5^\circ + \frac{1}{2}\sin^2 22.5^\circ$$

$$0.2500 \le 0.1464$$

CHSH Inequality

Considering the imperfections in experiments, generalized Bell equality — CHSH equality,

$$S = \left| E(\phi_A \phi_B) - E(\phi_A \phi_B') + E(\phi_A' \phi_B) + E(\phi_A' \phi_B') \right|$$

Local Reality prediction: $S_{MAX} \leq 2$

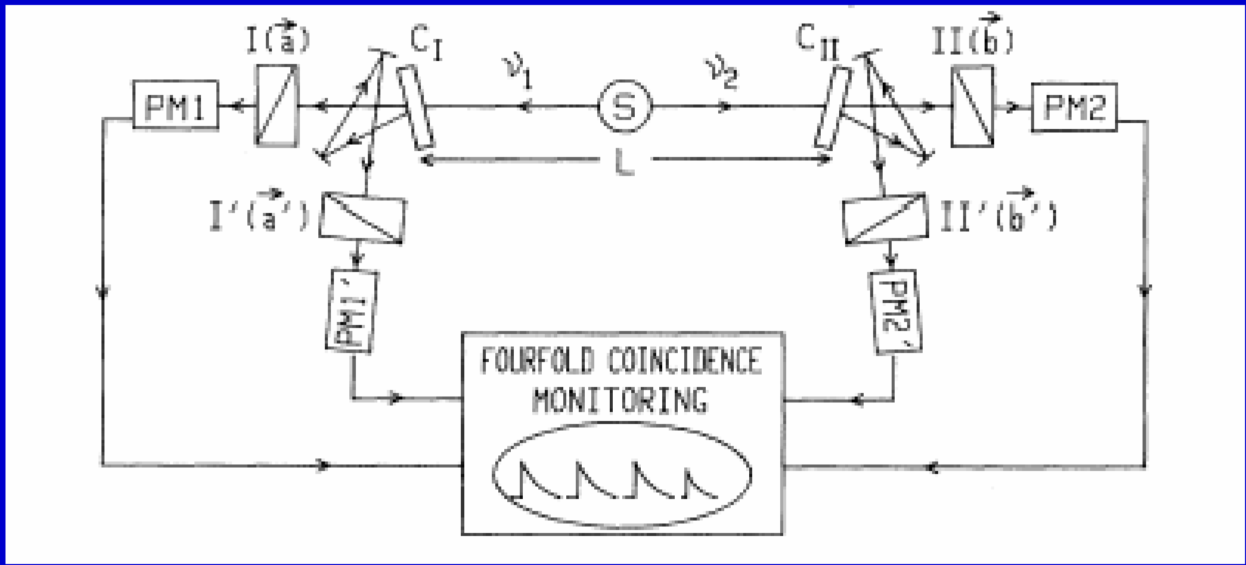
Quantum Mechanical prediction: $S_{MAX} = 2\sqrt{2}$

$$(\phi_A, \phi_A', \phi_B, \phi_B') = (0^\circ, 45^\circ, 22.5^\circ, 67.5^\circ)$$

J. Clauser et al., Phys. Rev. Lett. 23, 880 (1969)

Experimental Test of Bell Inequality

A. Aspect et al., Phys. Rev. Lett. 49, 1804 (1982)

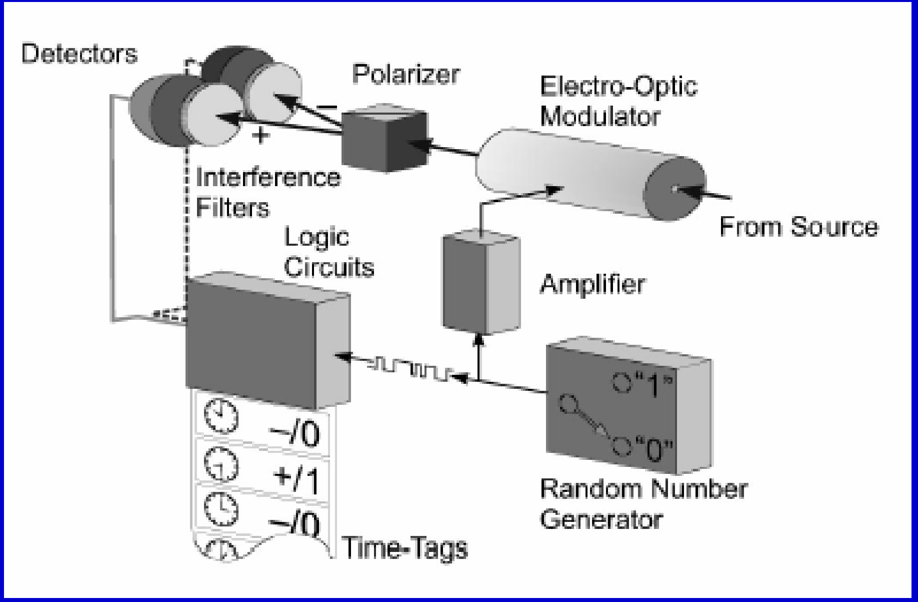


$S_{\exp} = 0.101 \pm 0.020$ violates a generalized inequality $S \leq 0$ by 5 standard deviations.

Drawbacks: 1. locality loop hole
2. detection loop hole

Experimental Test of Bell Inequality

G. Weihs et al., Phys. Rev. Lett. 81, 5039 (1998)



$S_{\exp} = 2.73 \pm 0.02$ violates CHSH inequality $S \le 2$

by 30 standard deviations.

Drawback: detection loop hole