

Lecture Note 3

Quantum Dense Coding and
Quantum Teleportation

Jian-Wei Pan

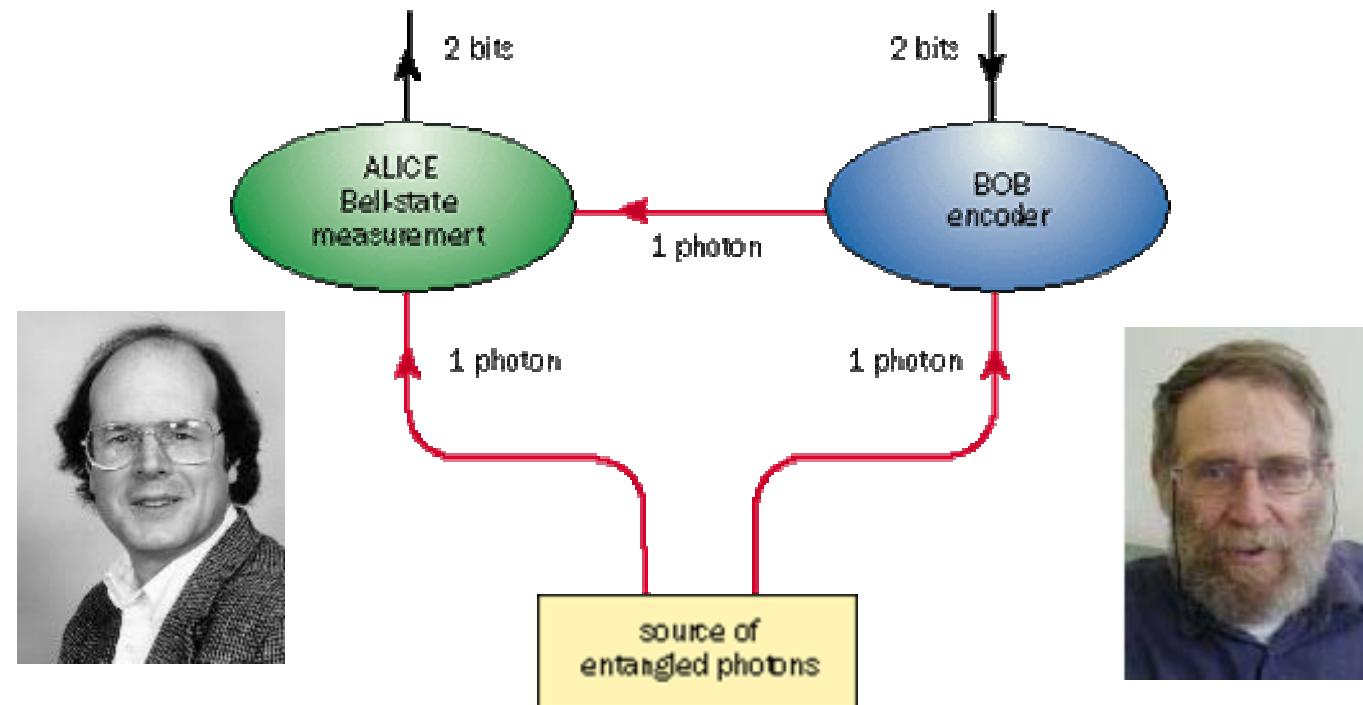
Dense Coding

Bell states – maximally entangled states:

$$|\Phi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 + |V\rangle_1|V\rangle_2)$$

$$|\Phi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 - |V\rangle_1|V\rangle_2) = \hat{\sigma}_{1z} |\Phi^+\rangle_{12}$$

$$|\Psi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2) = \hat{\sigma}_{1x} |\Phi^+\rangle_{12} \quad |\Psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2) = -i\hat{\sigma}_{1y} |\Phi^+\rangle_{12}$$



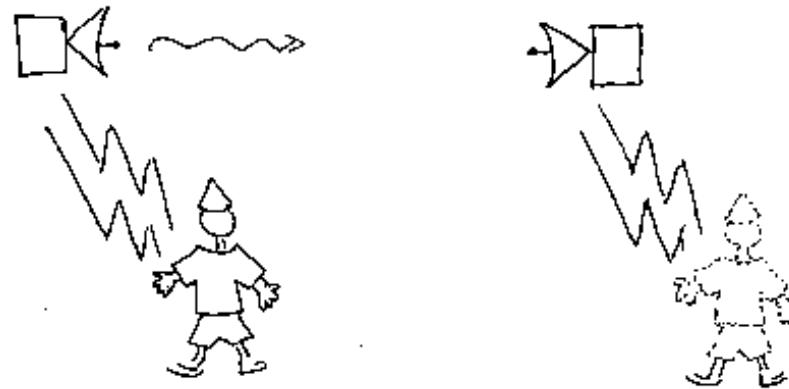
Theory:

[C. H. Bennett & S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992)]

Dense Coding

1. Alice and Bob share an entangled photon pair in the state of $|\Phi^+\rangle$.
2. Bob chooses one of the four unitary transformation $\{I, \sigma_z, \sigma_x, \sigma_y\}$ on his photon. The information of which choice is 2 bit.
3. Bob sends his photon to Alice.
4. Alice performs a joint Bell-state measurement on the photon from Bob and her photon.
5. With the measurement result, she can tell Bob's unitary transformation and achieve the 2 bit information.

Teleportation



- Classical Physics

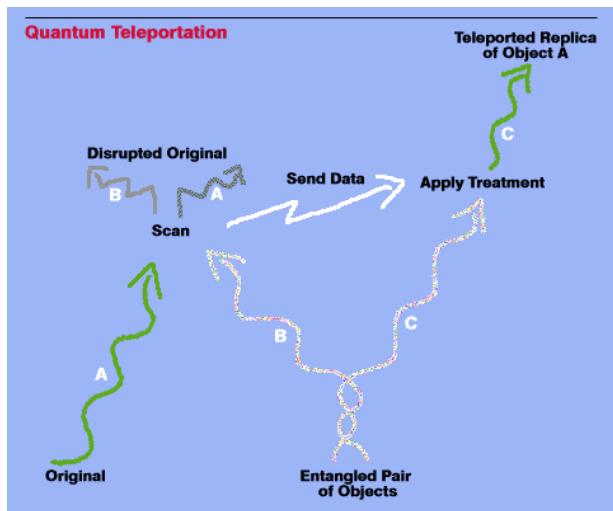
- Scanning and Reconstructing

- Quantum Physics

- Heisenberg Uncertainty Principle
Forbidden Extracting All the Information
from An Unknown Quantum State

Quantum Teleportation

--- “Six Author Scheme”



$$|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle|1\rangle \pm |1\rangle|0\rangle)$$

$$|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle \pm |1\rangle|1\rangle)$$

Bell states – maximally entangled states



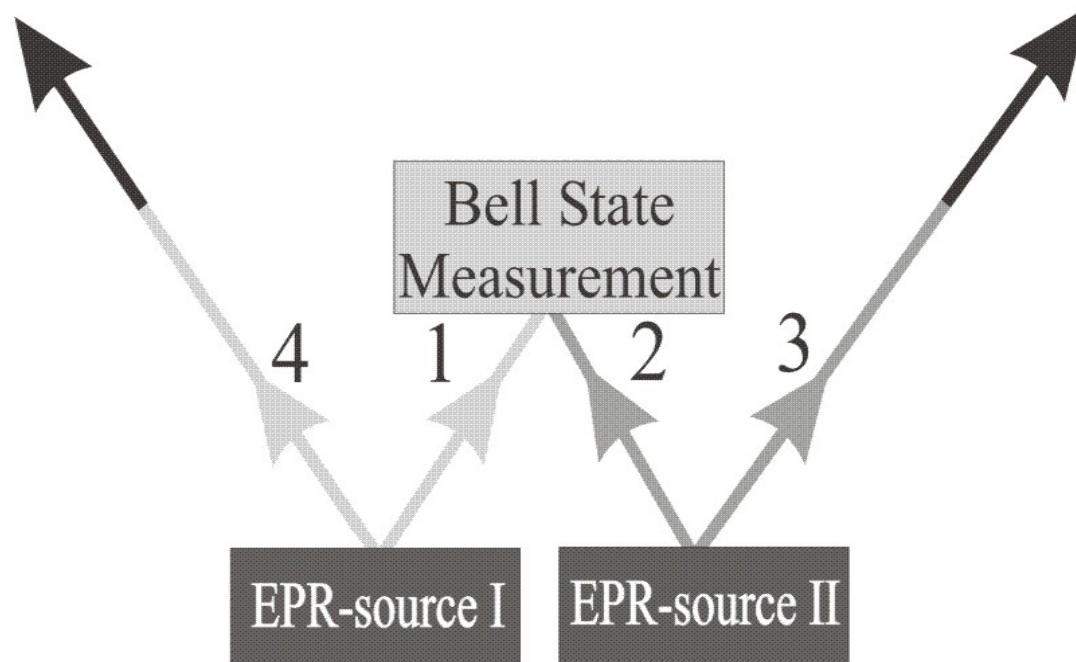
(top, left) Richard Jozsa, William K. Wootters, Charles H. Bennett. (bottom, left) Gilles Brassard, Claude Crépeau, Asher Peres. Photo: André Berthiaume.

$$\begin{aligned} |\Phi\rangle_A &= \alpha|0\rangle_A + \beta|1\rangle_A \quad (|\alpha|^2 + |\beta|^2 = 1) \\ |\Psi\rangle_{ABC} &= |\Phi\rangle_A \otimes |\Phi^+\rangle_{BC} \\ &= |\Phi^+\rangle_{AB} \otimes (\alpha|0\rangle_C + \beta|1\rangle_C) \\ &\quad + |\Phi^-\rangle_{AB} \otimes (\alpha|0\rangle_C - \beta|1\rangle_C) \\ &\quad + |\Psi^+\rangle_{AB} \otimes (\alpha|1\rangle_C + \beta|0\rangle_C) \\ &\quad + |\Psi^-\rangle_{AB} \otimes (\alpha|1\rangle_C - \beta|0\rangle_C) \end{aligned}$$

[C.H. Bennett et al., Phys. Rev. Lett. 73, 3801 (1993)]

Teleportation of entanglement

----Entanglement Swapping



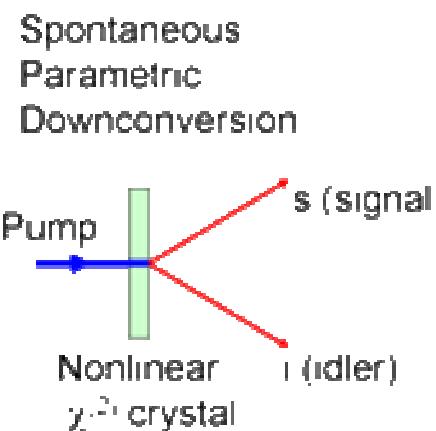
$$\begin{aligned} |\Psi\rangle_{1234} &= |\Phi^+\rangle_{14}\otimes|\Phi^+\rangle_{23} \\ &= |\Phi^+\rangle_{12}\otimes|\Phi^+\rangle_{34} + |\Phi^-\rangle_{12}\otimes|\Phi^-\rangle_{34} + \\ &\quad |\Psi^+\rangle_{12}\otimes|\Psi^+\rangle_{34} + |\Psi^-\rangle_{12}\otimes|\Psi^-\rangle_{34} \end{aligned}$$

[M. Zukowski et al., Phys. Rev. Lett. 71, 4287 (1993)]

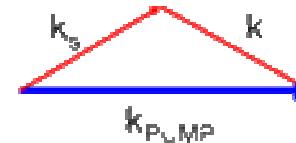
Experimental Ingredient

- Entangled photon pair
 - by Spontaneous Parametric down conversion (SPDC)
 - time bin entanglement
 - momentum entanglement
 - polarization entanglement
 -
- Bell state analyzer
photon statistics at a beamsplitter

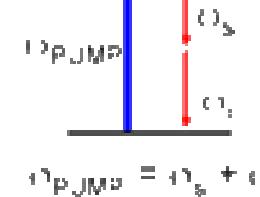
SPDC



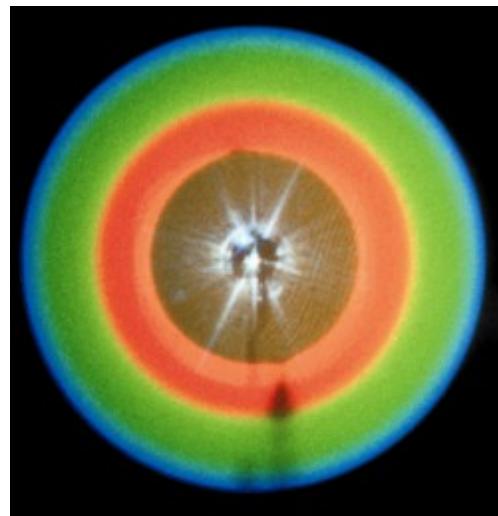
Momentum Conservation



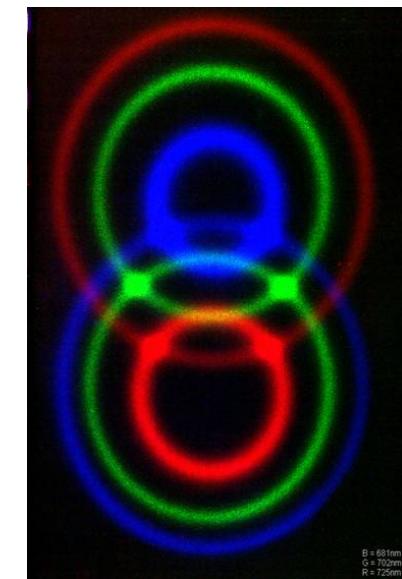
Energy conservation



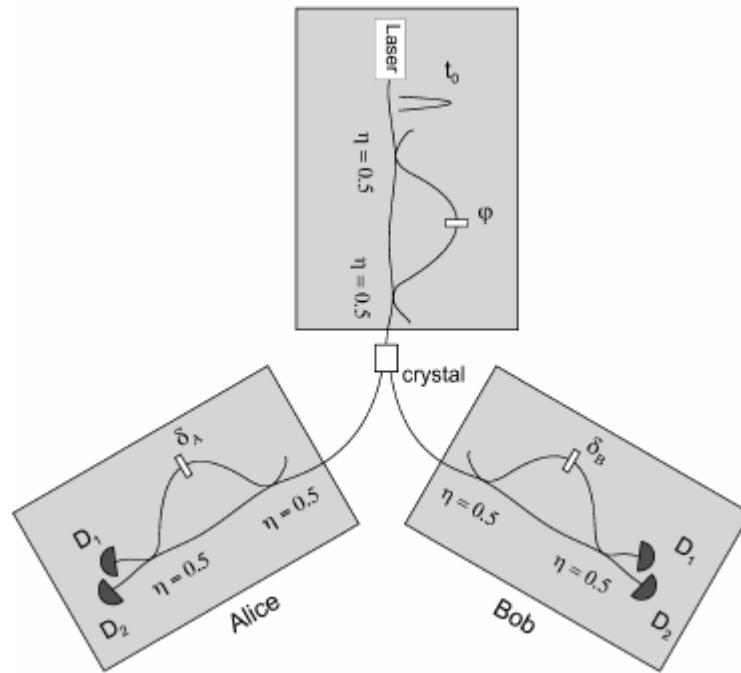
Type I



Type II



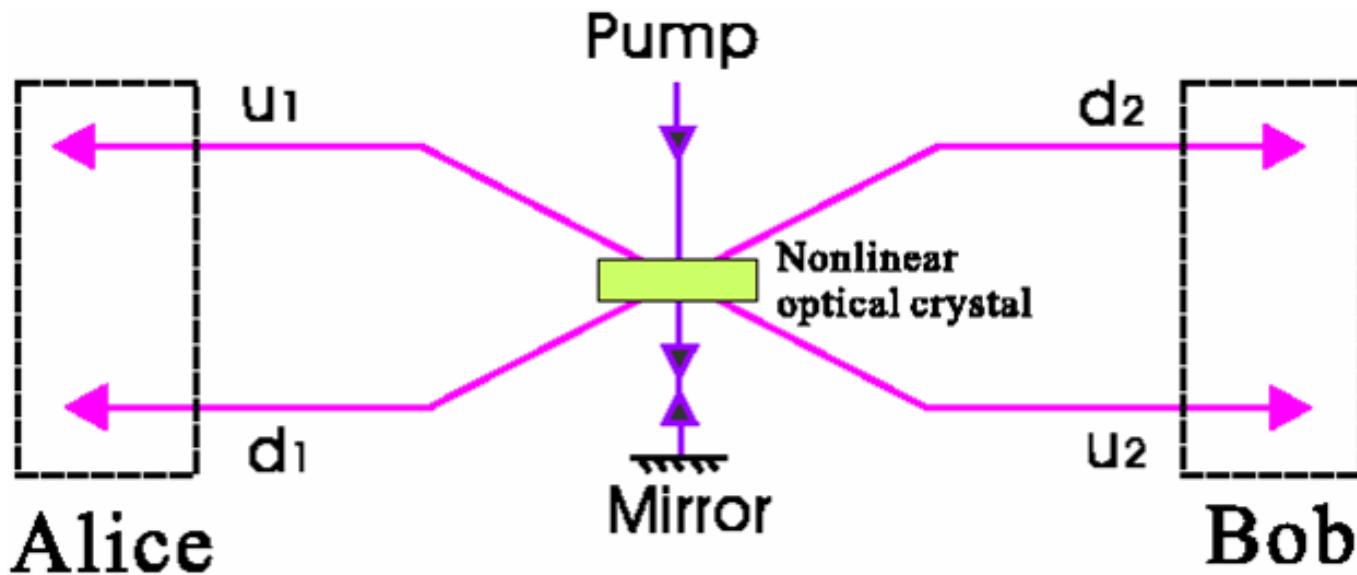
SPDC—time bin entanglement



$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|short\rangle_a |short\rangle_b + |long\rangle_a |long\rangle_b)$$

[J. Brendel et al., Phys. Rev. Lett. 82, 2594 (1999)]

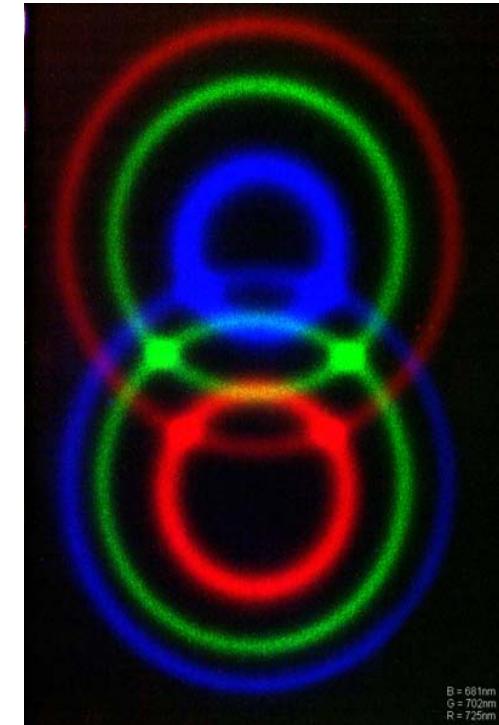
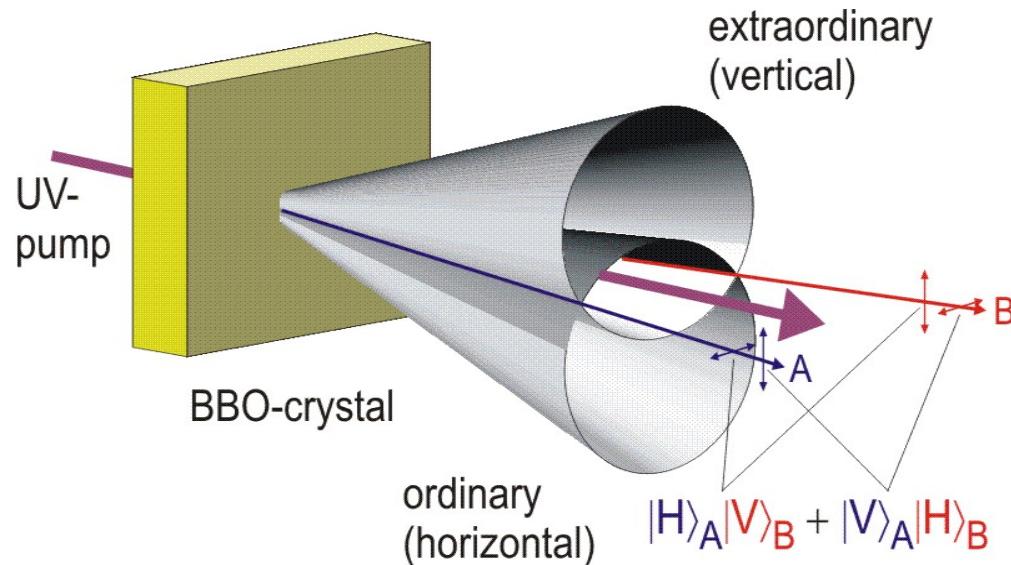
SPDC—momentum entanglement



$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|u\rangle_1 |d\rangle_2 + |d\rangle_1 |u\rangle_2)$$

[Z. B. Chen et al., Phys. Rev. Lett. 90, 160408 (2003)]

SPDC—polarization entanglement

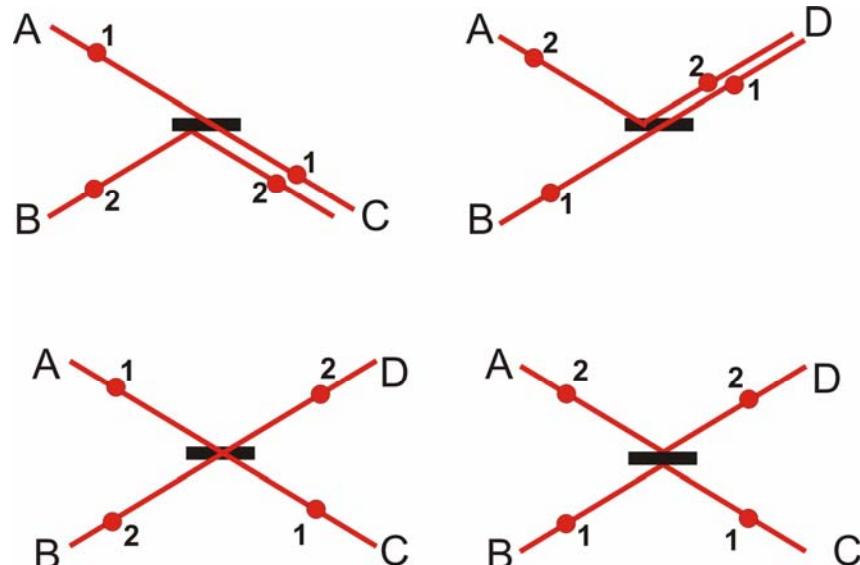
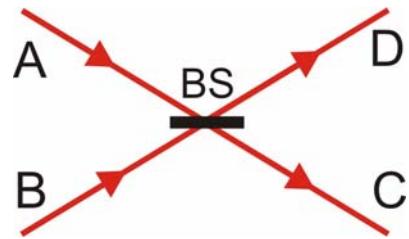


$$|\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2)$$

$$|\Psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 \pm |V\rangle_1|H\rangle_2)$$

[P. G. Kwiat et al., Phys. Rev. Lett. 75, 4337 (1995)]

Bell state analyzer with Linear Optics



Controlled-NOT gate

$$HH \rightarrow HV, \quad HV \rightarrow HH$$

$$VH \rightarrow VH, \quad VV \rightarrow VV$$

$$(H + V)H \rightarrow HV + VH$$

$$|\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|H\rangle_2 \pm |V\rangle_1|V\rangle_2)$$

$$|\Psi^+\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2)$$

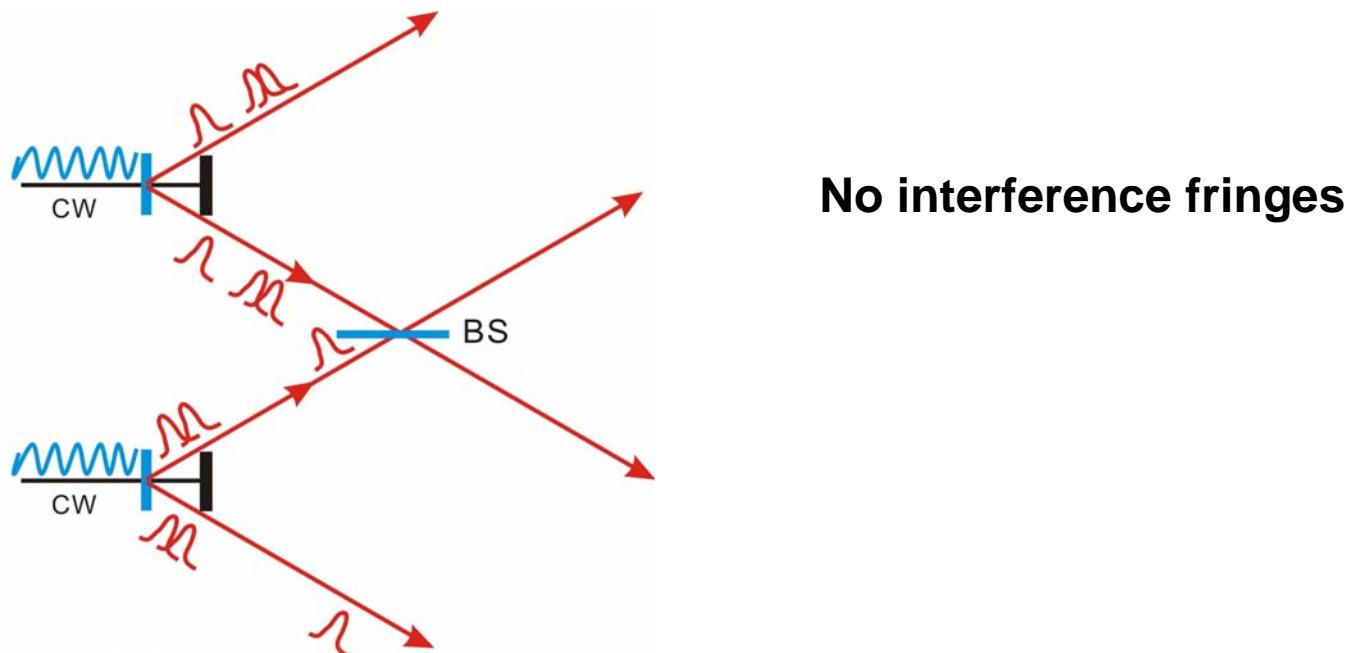
$$|\Psi^-\rangle_{12} = \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2)$$

[H. Weinfurter, *Europhys. Lett.* 25, 559 (1995)]

[J.-W. Pan et al., *Phys. Rev. A* (1998)]

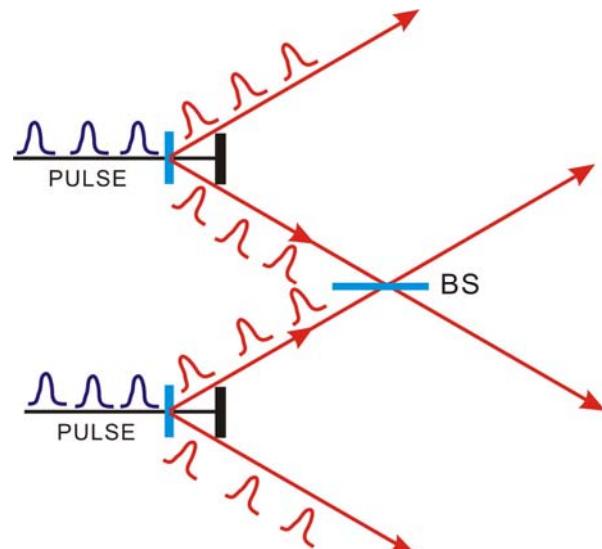
Bell state analyzer with Linear Optics

- To realize a Bell state measurement is to make the two input SPDC photons indistinguishable on the BS. The method is to make the two photons spatially and timely overlapped on the BS perfectly.
- However, the timely overlap is difficult since the SPDC photon has a ultrasmall coherent time about 100fs, definitely shorter than the time resolution of state-of-the-art single photon detector.
- What we can do is to scan the interference fringes to make it sure that the two photons arrive at the same time. But a cw laser will...

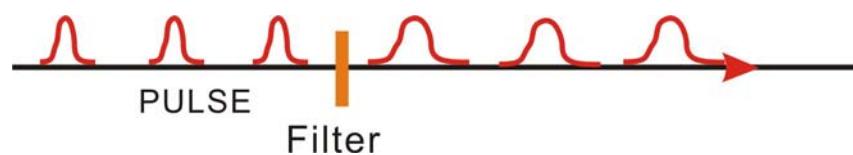


Bell state analyzer with Linear Optics

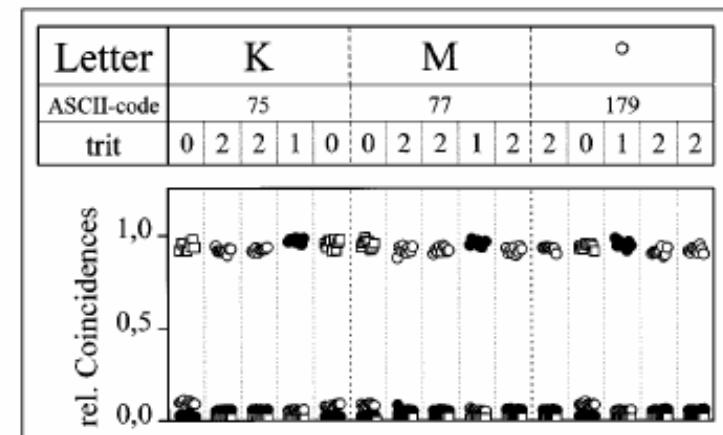
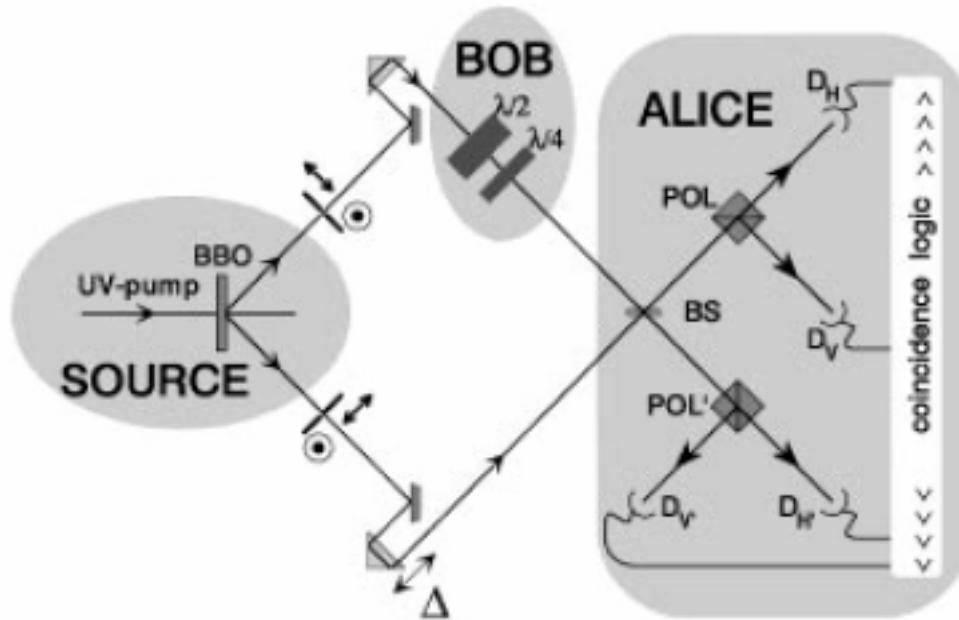
- A solution is to use Pulse laser



The pulse will bring some time jitter to the SPDC photon, we insert a narrow band filter can extend the coherent time of the SPDC photon



Experimental Dense Coding



$$\begin{aligned} 0 &\equiv |\Phi^-\rangle \hat{=} \square \\ 1 &\equiv |\Psi^+\rangle \hat{=} \bullet \\ 2 &\equiv |\Psi^-\rangle \hat{=} \circ \end{aligned}$$

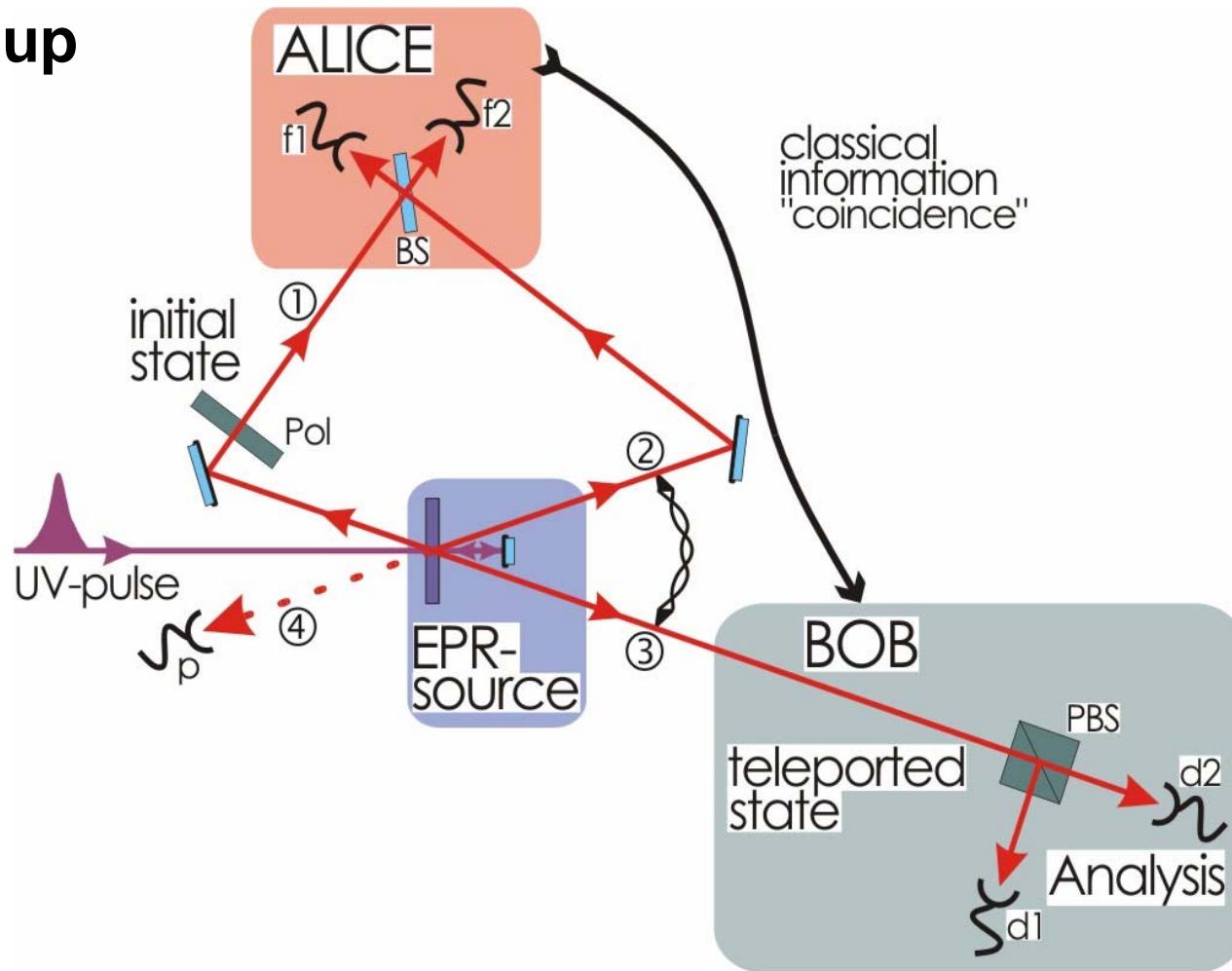
[K. Mattle et al., Phys. Rev. Lett. 76, 4656 (1996)]

Experimental Realizations of quantum teleportation

- ◆ D. Bouwmeester, et al., **Nature** 390, 575-579 (1997) (photons)
- ◆ D. Boschi, et al., **Phys. Rev. Lett.** 80, 1121-1125 (1998) (photons).
- ◆ J-W. Pan, et al., **Phys. Rev. Lett.** 80, 3891–3894 (1998) (mixed state of photons).
- ◆ A. Frusawa, et al., **Science** 282, 706 (1999) (continuous-variable)
- ◆ M. Riebe, et al., **Nature** 429, 734-737 (2004) (trapped calcium ions).
- ◆ M.D. Barret, et al., **Nature** 429, 737-739 (2004) (trapped beryllium ions)
- ◆ I. Marcikic, et al., **Nature** 421, 509-513 (2003) (long distance)
- ◆ R. Ursin, et al., **Nature** 430, 849 (2004) (long distance)
- ◆ Z. Zhao, et al., **Nature** 430, 54 (2004) (open destination teleportation)

Experimental Quantum Teleportation

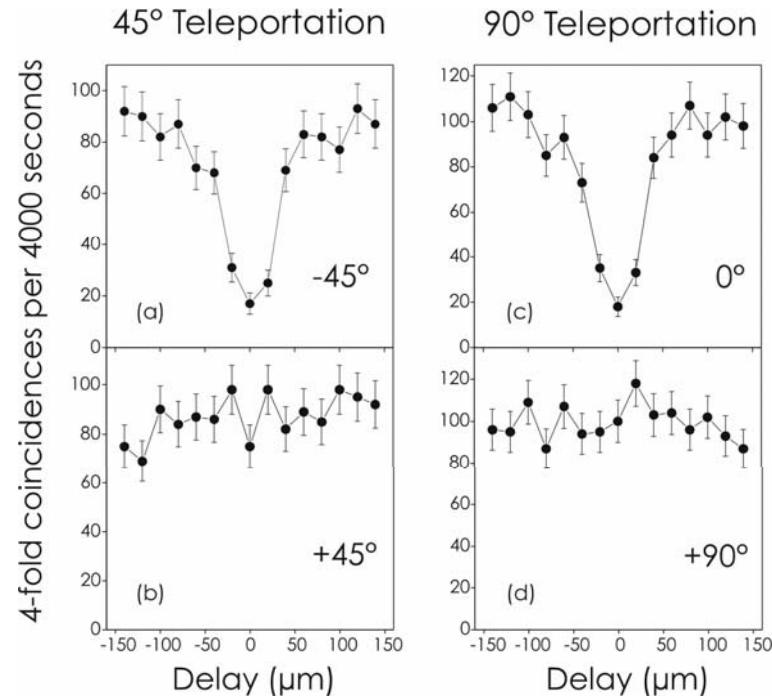
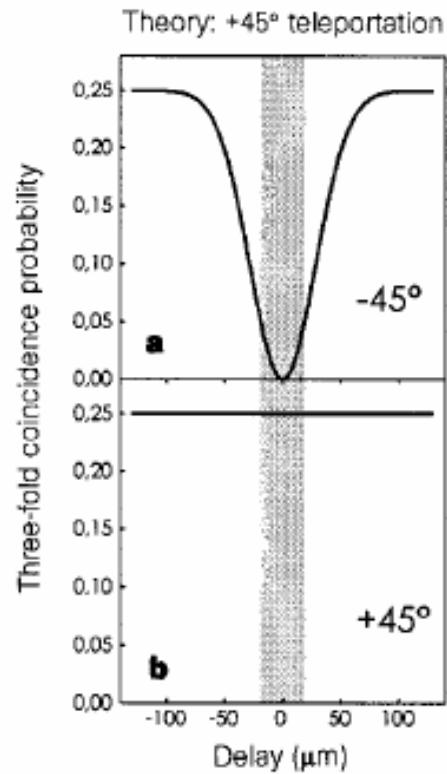
The setup



[D. Bouwmeester et al., Nature 390, 575 (1997)]

Experimental Quantum Teleportation

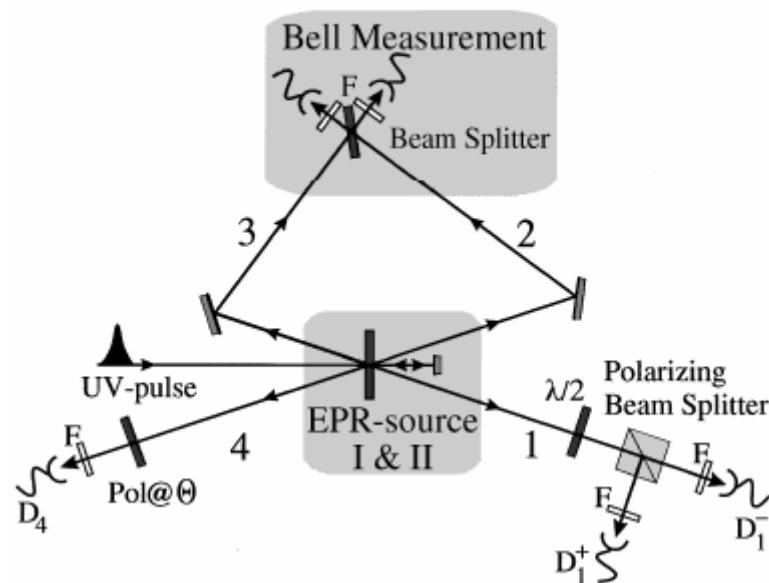
The result



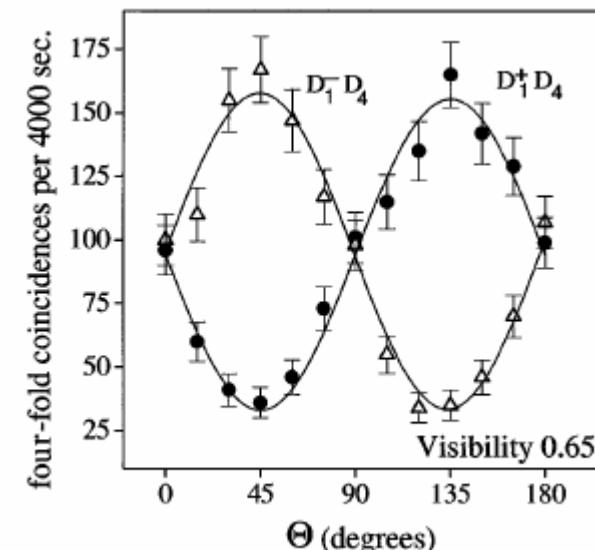
[D. Bouwmeester et al., Nature 390, 575 (1997)]

Experimental Entanglement Swapping

The setup

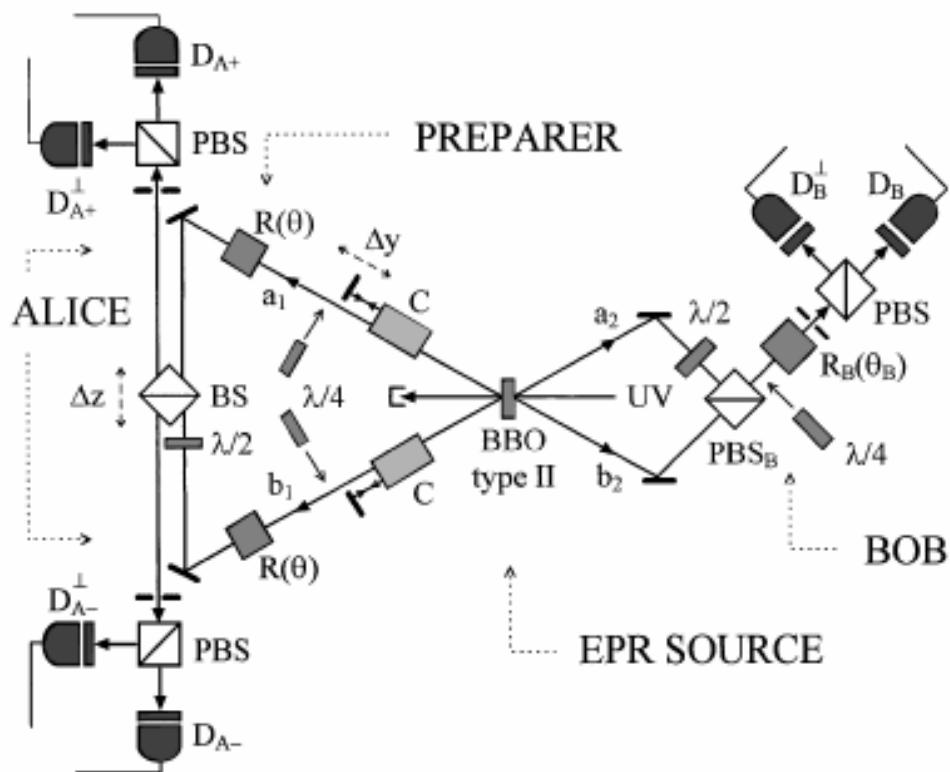


The result



[J.-W. Pan et al., Phys. Rev. Lett. 80, 3891 (1998)]

A Two-Particle Quantum teleportation experiment



[D. Boschi et al., Phys. Rev. Lett. 80, 1121 (1998)]

1. Sharing EPR Pair

$$|\psi\rangle_1 = \alpha|H\rangle_1 + \beta|V\rangle_1$$

2. Initial State Preparation

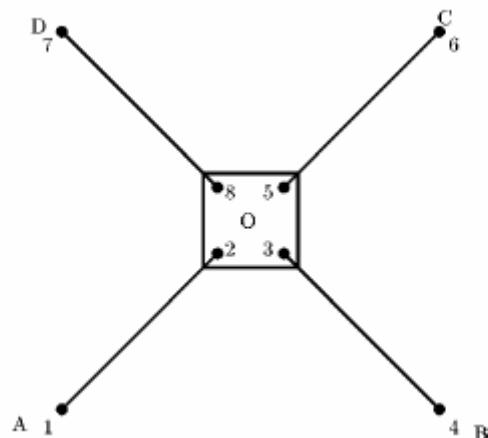
$$\begin{aligned} |\Psi^+\rangle &= \frac{1}{\sqrt{2}}(|H\rangle_1|V\rangle_2 + |V\rangle_1|H\rangle_2) \\ &\rightarrow \frac{1}{\sqrt{2}}(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)|H\rangle_1|V\rangle_2 \end{aligned}$$

3. BSM

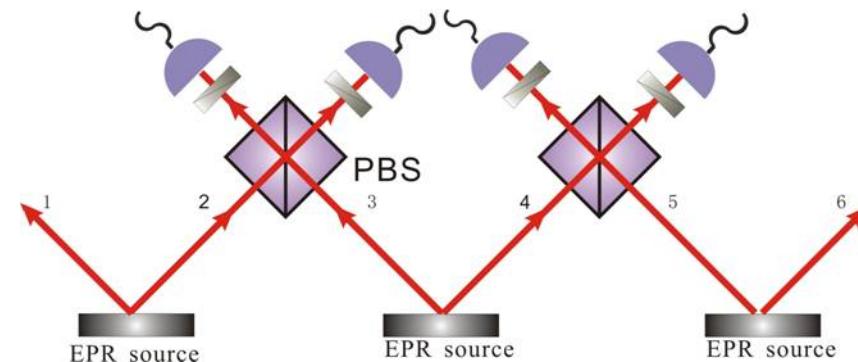
$$\begin{aligned} |\Phi\rangle &= \frac{1}{\sqrt{2}}(|a_1\rangle|a_2\rangle + |b_1\rangle|b_2\rangle)|\psi\rangle_1|V\rangle_2 \\ &= \frac{1}{2}[|a_1\rangle|V\rangle_1 + |b_1\rangle|H\rangle_1)(\beta|a_2\rangle + \alpha|b_2\rangle)|V\rangle_2 \\ &\quad + (|a_1\rangle|V\rangle_1 - |b_1\rangle|H\rangle_1)(\beta|a_2\rangle - \alpha|b_2\rangle)|V\rangle_2 \\ &\quad + |a_1\rangle|H\rangle_1 + |b_1\rangle|V\rangle_1)(\alpha|a_2\rangle + \beta|b_2\rangle)|V\rangle_2 \\ &\quad + |a_1\rangle|V\rangle_1 - |b_1\rangle|H\rangle_1)(\alpha|a_2\rangle - \beta|b_2\rangle)|V\rangle_2] \end{aligned}$$

Applications of Entanglement Swapping

Quantum telephone exchange

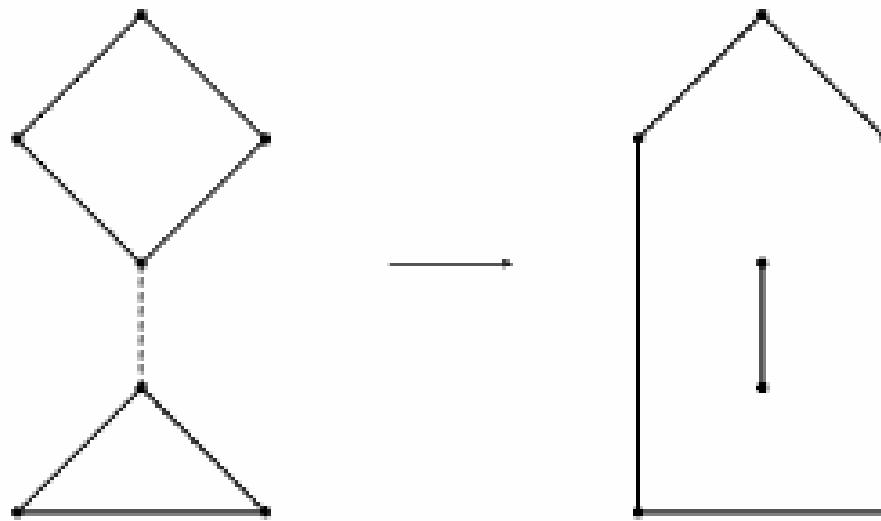


Speed up the distribution of entanglement



[S. Bose et al., Phys. Rev. A 57, 822 (1998)]

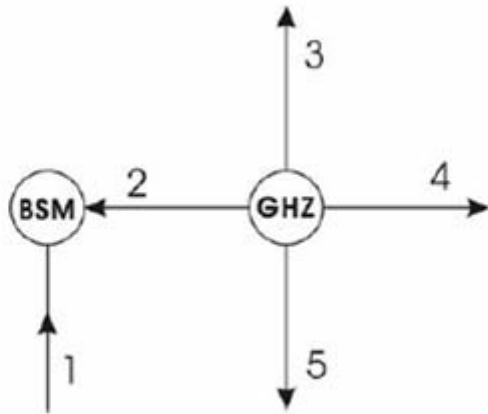
Applications of Entanglement Swapping



$$|E(N)\rangle \otimes |E(3)\rangle \xrightarrow{BSM} |E(N+1)\rangle \otimes |E(2)\rangle$$

[S. Bose et al., Phys. Rev. A 57, 822 (1998)]

Open-Destination Teleportation



Sharing a secret quantum state
of single particles

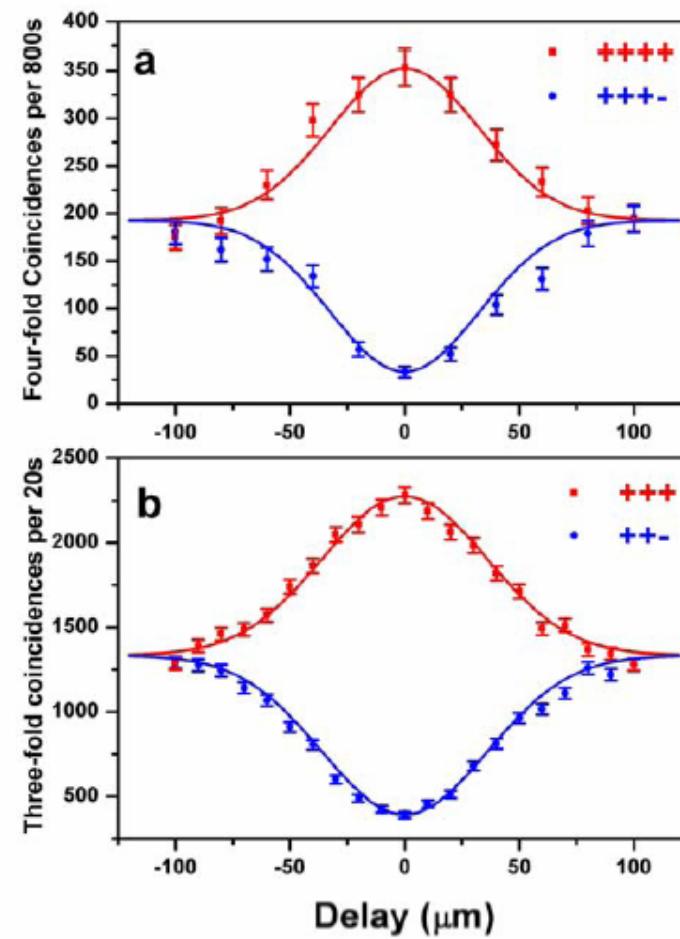
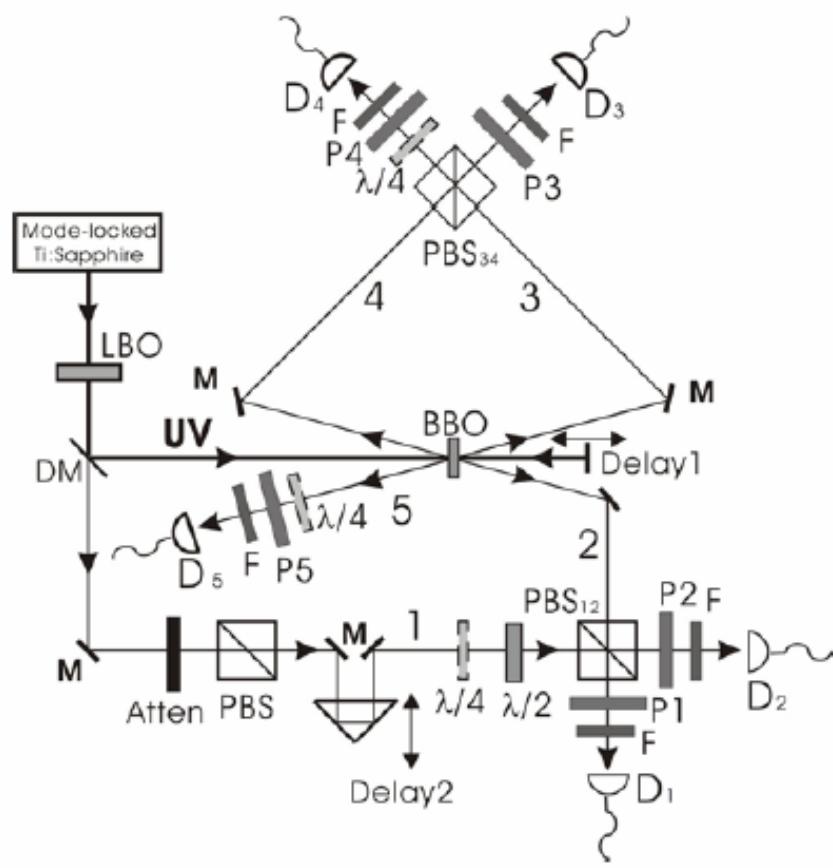
$$|\Psi\rangle_1 = \alpha |H\rangle_1 + \beta |V\rangle_1$$

$$\begin{aligned} |\Psi\rangle_{12345} &= |\Psi\rangle_1 |\Phi\rangle_{2345} \\ &= \frac{1}{2} [|\Phi^+\rangle_{12} (\alpha |H\rangle_3 |H\rangle_4 |H\rangle_5 + \beta |V\rangle_3 |V\rangle_4 |V\rangle_5) \\ &\quad + |\Phi^-\rangle_{12} (\alpha |H\rangle_3 |H\rangle_4 |H\rangle_5 - \beta |V\rangle_3 |V\rangle_4 |V\rangle_5) \\ &\quad + |\Psi^+\rangle_{12} (\alpha |V\rangle_3 |V\rangle_4 |V\rangle_5 + \beta |H\rangle_3 |H\rangle_4 |H\rangle_5) \\ &\quad + |\Psi^-\rangle_{12} (\alpha |V\rangle_3 |V\rangle_4 |V\rangle_5 - \beta |H\rangle_3 |H\rangle_4 |H\rangle_5)] \end{aligned}$$

[A. Karlsson et al., Phys. Rev. A 58, 4394 (1998)]

[R. Cleve et al., Phys. Rev. Lett. 83, 648 (1999)]

Experimental Setup



Experimental Results

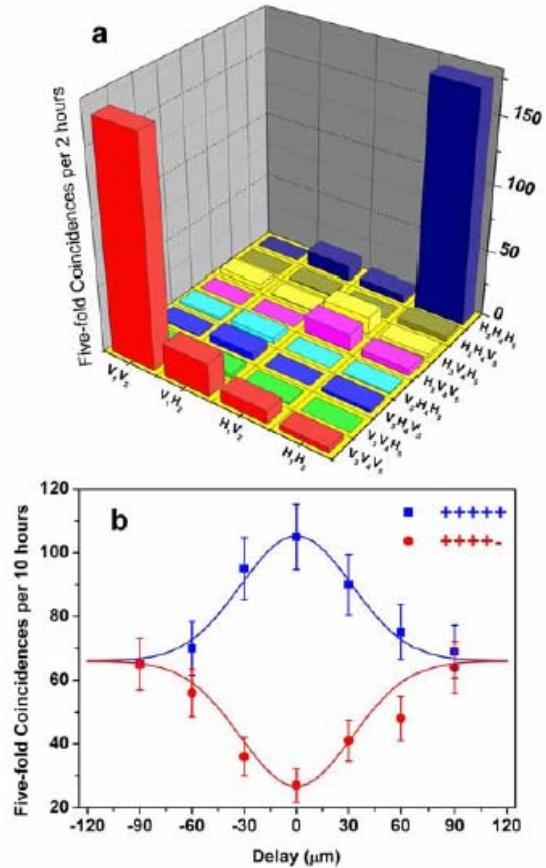


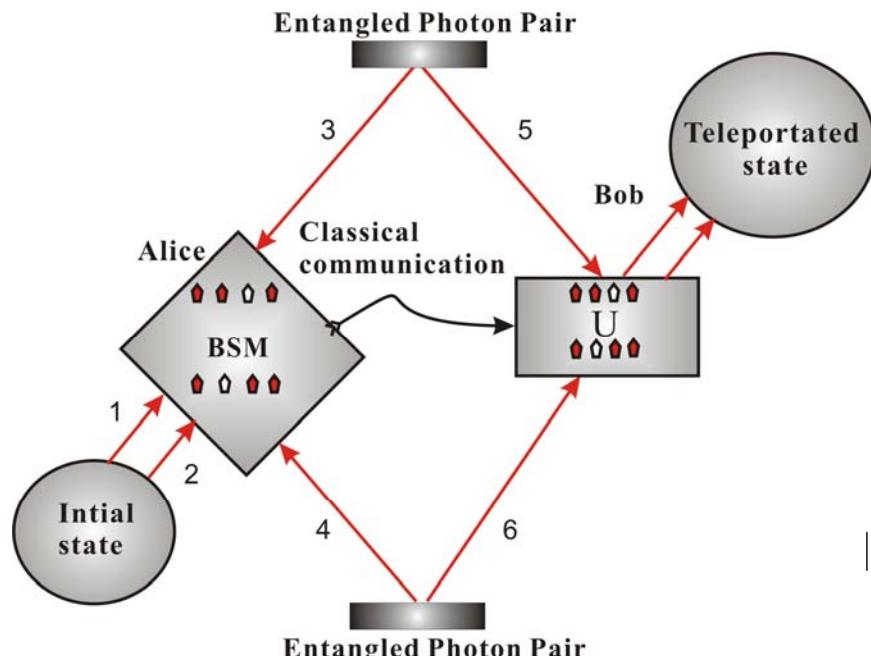
Table 1. Fidelities of open-destination teleportation at two different locations

polarization	Fidelities	
	Location 4	Location 5
$ +\rangle$	0.79 ± 0.04	0.80 ± 0.03
$ -\rangle$	0.81 ± 0.04	0.77 ± 0.04
$ R\rangle$	0.75 ± 0.04	0.79 ± 0.04
$ L\rangle$	0.79 ± 0.04	0.82 ± 0.03

[Z. Zhao et al., Nature 430, 54 (2004)]

Teleportation of a composite system

----Scheme



Initial State

$$|\chi\rangle_{34} = \alpha|H\rangle_3|H\rangle_4 + \beta|H\rangle_3|V\rangle_4 + \gamma|V\rangle_3|H\rangle_4 + \delta|V\rangle_3|V\rangle_4$$

H---horizontal polarization

V--- vertical polarizations

Just as the single qubit teleportation, first teleport photon 1 to photon 5

$$|\chi\rangle_{12}|\Phi^+\rangle_{35} = \frac{1}{2}(|\Phi^+\rangle_{13}|\chi\rangle_{52} + |\Phi^-\rangle_{13}\hat{\sigma}_z|\chi\rangle_{52} + |\Psi^+\rangle_{13}\hat{\sigma}_x|\chi\rangle_{52} + |\Psi^-\rangle_{13}(-i\hat{\sigma}_y|\chi\rangle_{52}))$$

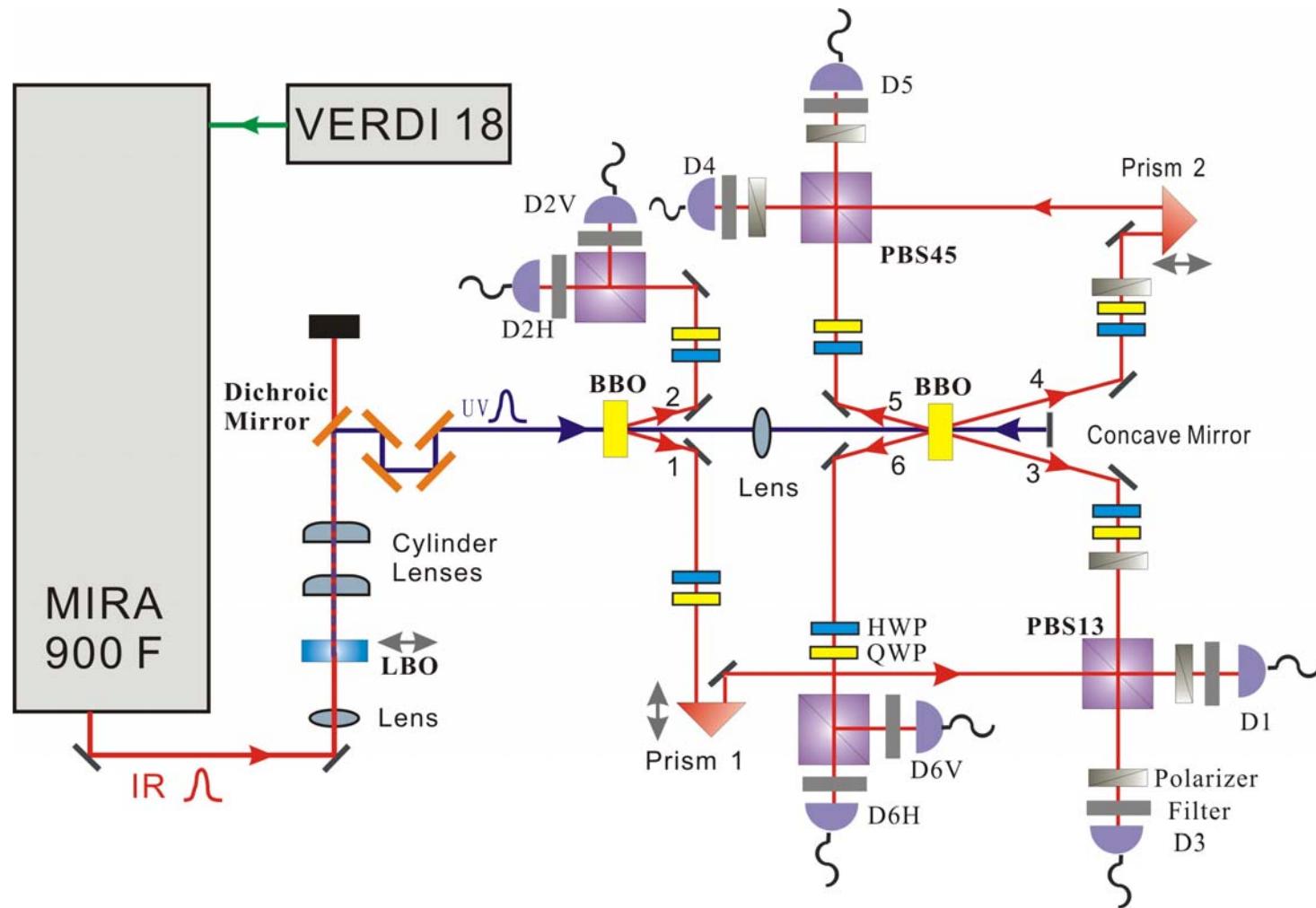
And then, we teleport photon 2 to photon 6

$$|\chi\rangle_{52}|\Phi^+\rangle_{46} = \frac{1}{2}(|\Phi^+\rangle_{24}|\chi\rangle_{56} + |\Phi^-\rangle_{24}\hat{\sigma}_z|\chi\rangle_{56} + |\Psi^+\rangle_{24}\hat{\sigma}_x|\chi\rangle_{56} + |\Psi^-\rangle_{24}(-i\hat{\sigma}_y|\chi\rangle_{56}))$$

Finally, we can teleport the state of photon 1,2 to photon 5,6

Teleportation of a composite system

----Setup



Teleportation of a composite system

----Result

State	Fidelity	Fidelity with noise reduction
HV	0.86 ± 0.03	0.97 ± 0.03
HV-VH	0.60 ± 0.03	0.71 ± 0.03
(H+V)(H-iV)	0.75 ± 0.02	0.83 ± 0.02
Average	0.74 ± 0.03	0.84 ± 0.03

Fidelity:

Pure state

$$F = \left| \langle \psi | \psi \rangle_{\text{Tel}} \right|^2$$

Mixed state

$$F = \text{Tr}(\rho | \psi \rangle_{\text{ini}} \langle \psi |)$$

Well beyond the
clone limit 0.40

[A. Hayashi et al., Phys. Rev. A. 72, 032325 (2005).]

Teleported State of HV-VH is Still Entangled!

$$\begin{aligned} \text{Tr}(\rho W) &= \frac{1}{2} \text{Tr}[\rho (|HH\rangle\langle HH| + |VV\rangle\langle VV| + |++\rangle\langle ++| \\ &\quad + |--\rangle\langle --| - |RL\rangle\langle RL| - |LR\rangle\langle LR|)] \\ &= -0.23 \pm 0.04 < 0 \end{aligned}$$

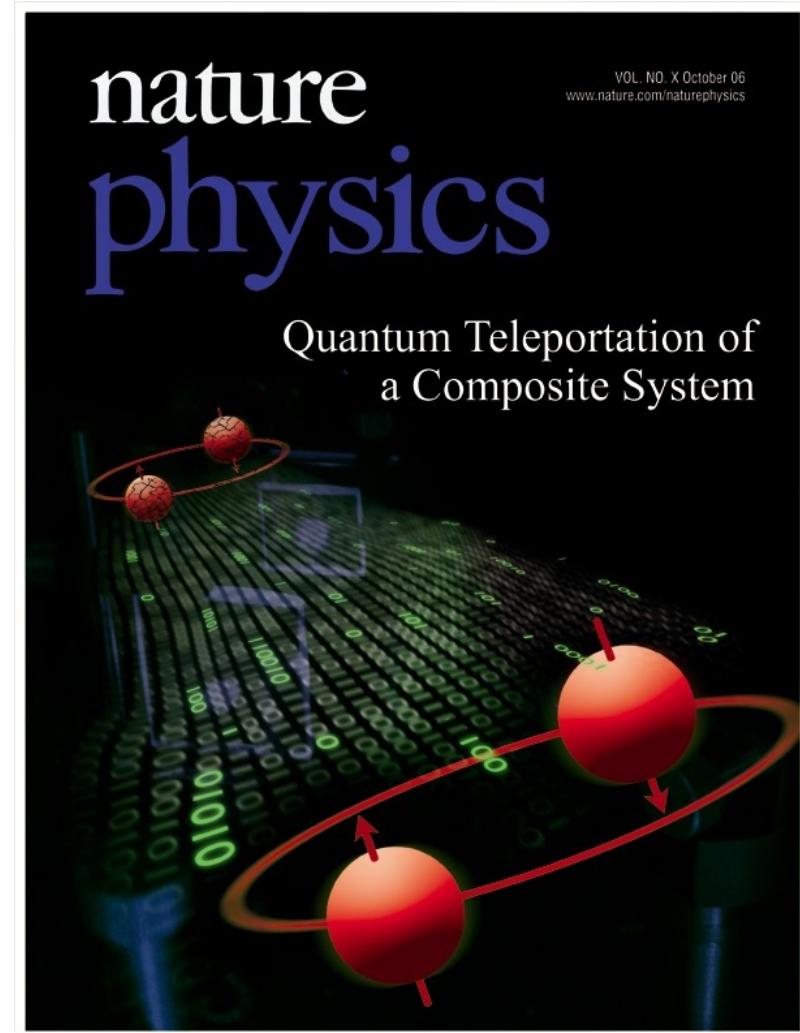
[M. Barbieri, et al. Phys. Rev. Lett. 91, 227901 (2003).]

Experimental Teleportation of a Two-Qubit Composite System

Q. Zhang et al.,

Experimental quantum teleportation
of a two-qubit composite system,

Nature Physics 2, 678-682 (2006).



Our dream

